Universal estimates for eigenvalues of Schrödinger operators on domains in Euclidean space and two-point homogeneous spaces

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Introduction

Several authors have worked on the problem of universal eigenvalue estimates for the Dirichlet Laplacian and for Schrödinger operators (see [1]–[5] for some of the main references and for the most recent results, further references can be found there). Although the result obtained in [1] is sharp for the one-dimensional harmonic oscillator the known results seem to be somewhat disappointing for the Dirichlet Laplacian (cf. the end of the introduction of [1]). We notice that the result obtained in [1], Proposition 6 for the one-dimensional harmonic oscillator includes a virial assumption which apparently cannot be obtained from the knowledge of the eigenvalues alone.

Our starting point is a simple calculation which gives a "sharp" upper bound for the second Dirichlet eigenvalue of a one-dimensional Schrödinger operator with nonnegative potential. The test function is obtained as a product of a ground state eigenfunction and a trigonometric monomial. This is in contrast to the approach in [1]–[5] where products by linear functions have been used (seemingly motivated by the position operator in quantum mechanics). This motivates our version of a trace formula. Thus, the problem of obtaining universal estimates for Dirichlet eigenvalues can be treated using harmonic analysis.

References

[1] Harrell, E. M., II; Stubbe, J.
On trace identities and universal eigenvalue estimates for some partial differential operators, Trans. Amer. Math. Soc. 349 (1997), 2037–2055

[2] Hile, G. N.; Protter, M. H.
Inequalities for eigenvalues of the Laplacian, Indiana Univ. Math. J. 29 (1980), 523–538

[3] Levitin, M.; Parnovski, L.
Commutators, spectral trace identities, and universal estimates for eigenvalues, J.Funct. Anal. 192 (2002), 425-445.

[4] Payne, L. E.; Pólya, G.; Weinberger, H. F.
On the ratio of consecutive eigenvalues, J. Math. and Phys. 35 (1956), 289–298

[5] Yang, Honcang
An estimate of the difference between consecutive eigenvalues, Preprint IC/91/60 of the Intl. Centre for Theoretical Physics (1991), Revised Preprint, from Academica Sinica (1995)

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