Smolyak's Algorithm and Function Spaces With Dominating Mixed Smoothness I

Tino Ullrich

In the first part of the talk we introduce Smolyak's algorithm in a very general setting. These algorithms are given by sums of certain tensor products of one dimensional algorithms. What we have in mind are algorithms, i.e. operators, acting on univariate spaces of periodic functions, e.g. Fourier partial sums, de la Vallée Poussin means and especially sampling operators. We give several equivalent representations of the algorithm, prove invariance properties and point out the connection to hyperbolic cross approximation. Furthermore, in case of sampling operators, we pay attention to the question, whether Smolyak's algorithm interpolates and prove assertions concerning the number of function values used depending on the dimension d.
The second part of the talk is devoted to Besov and Lizorkin-Triebel spaces of periodic functions with dominating mixed smoothness properties. Introduced from the Fourier analytical point of view we also give several characterizations by differences.

Smolyak's Algorithm and Function Spaces With Dominating Mixed Smoothness II

Tino Ullrich

This talk is about approximate recovery of functions with dominating mixed derivative. We consider several algorithms of Smolyak type (denoted by $A(m,d)$) and investigate the behavior of

$$\|Id - A(m,d) : F(\mathbb{T}^d) \to L_p(\mathbb{T}^d)\| \quad , \quad 1 \le p \le \infty \ ,$$

where $F(\mathbb{T}^d)$ denotes a class of periodic functions with dominating mixed derivative. We also make some remarks to best approximation with respect to hyperbolic crosses. But the main focus lies on the case of sampling operators and the approximate recovery by a finite number of points.