Abstract:

Let G be a semisimple algebraic group with Lie algebra $\mathfrak{g}$ and a Borel subgroup B. For a nilpotent G-orbit $\mathcal{O} \subset \mathfrak{g}$, let $d_{\mathcal{O}}$ denote the maximal dimension of a subspace $V \subset \mathfrak{g}$ that is contained in the closure of $\mathcal{O}$. We prove that $d_{\mathcal{O}} \leqslant \frac{1}{2} \dim \mathcal{O}$ and this upper bound is attained if and only if $\mathcal{O}$ is a Richardson orbit. Furthermore, if V is B-stable and $\dim V = \frac{1}{2} \dim \mathcal{O}$, then V is the nilradical of a polarisation of $\mathcal{O}$. Every nilpotent orbit closure has a distinguished B-stable subspace constructed via an $\mathfrak{sl}_2$-triple, which is called the *Dynkin ideal*.

We then characterise the nilpotent orbits $\mathcal{O}$ such that the Dynkin ideal

(1) has the minimal dimension among all B-stable subspaces $\mathfrak{c}$ such that $\mathfrak{c} \cap \mathcal{O}$ is dense in $\mathfrak{c}$, or

(2) is the only B-stable subspace $\mathfrak{c}$ such that $\mathfrak{c} \cap \mathcal{O}$ is dense in $\mathfrak{c}$.