Replicating Life Insurance Liabilities

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Solvency II is a new regulation for European insurance industry. It is expected to be implemented by the end of 2012. Important aspects of Solvency II are:

- **Risk based capital requirement**
  - Recommended risk measure is 99.5% VaR

- **Valuation of insurance liabilities**
  - Best Estimate of liabilities including future profit sharing
  - Need to include the valuation of options and guarantees

- **Risk management**
  - Effective asset liability management
  - Definition of expected dynamic management actions

- **Requirements for transparency**
Best estimate liability cash flows of P&C Insurers do not depend on capital market.

Assets and liabilities can be stressed independently to determine the economic risk capital and solvency ratio.
Future profits + Future bonuses = Assets – Equity – Guaranteed liabilities

- Assets, Equity and guaranteed benefits can usually be valued analytically
- The value of future share holders profits and future policy holders bonuses needs to be determined by stochastic simulation

\[ MCEV = ANW + PVFP - CoC \]
What should be modelled

- Projection over the life time of the insurance policy
  - At least 30 years
- Plain vanilla and exotic instruments in insurers portfolio
  - Strait bonds, shares, options, swaptions, (multi-) callable bonds, CMS-floaters (with floor, cap), etc.
- Tax
- Regulatory constraints
  - Guaranteed bonuses (with lock-in effect)
  - Profit sharing restrictions
  - Solvability requirements
  - Etc.
- Management actions
  - Bonus distribution
  - Investment and disinvestment rules
  - Etc.
- Rational policy holder behavior
- ...

NO ANALYTICAL SOLUTION
Replicating Life Insurance Liabilities

**Theory:**

\[ PVFP(t) = \text{Num}(t) E \left( \sum_{i=0}^{n} CF(t_i, \omega) \bigg/ \text{Num}(t_i, \omega) \bigg| F_t \right) \]

- The required complexity can not be handled analytically in praxis. The only way to access the company value and risks is by Monte-Carlo simulation.
- Generate the shareholders cash flows resulting from the life insurance business in market consistent Monte-Carlo scenarios over the life time of insurance contracts. Take the average of the cash flows discounted with a chosen numeraire.

**Praxis:**

\[ PVFP(t_0) = \frac{1}{N} \sum_{k=1}^{N} \sum_{i=0}^{n} CF(t_i, \text{Scenario}_k) / \text{Num}(t_i, \text{Scenario}_k) \]
Computational complexity under Solvency II

<table>
<thead>
<tr>
<th>Task</th>
<th>Complexity</th>
<th>Computational Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCEV Calculation</td>
<td>Projection over 40 years</td>
<td>Several hours</td>
</tr>
<tr>
<td>MCEV-Sensitivities (app. 20 Runs)</td>
<td>Projection over 40 years for each sensitivity MCEV calculation</td>
<td>Several hours per sensitivity</td>
</tr>
</tbody>
</table>
| Economic Risk Capital (ERC)         | • 1-year Projection with 10000 „Best Estimate“ Scenarios  
• Calibration of the market model and scenario generation for MCEV calculation  
• Projection over 40 years for each „Best Estimate“ Scenario | Several years!!! |

Direct calculation of economic risk capital requires nested simulation. It can not be performed within a reasonable time.
Approximating MCEV

- **Delta-Gamma Approach**
  - Quadratic approximation of market sensitivities

- **Curve fitting**
  - Find a curve (usually a low degree polynomial function) that interpolates computed sensitivities

- **Least Squares Monte-Carlo**
  - Regression of Monte-Carlo values onto a set of basis functions
  - Motivated by Longstaff-Schwartz algorithm for Monte-Carlo pricing of exotic options

- **Replicating portfolios**
  - Finding a (static) portfolio that replicates MCEV (or Liabilities)
  - It should not be confused with dynamical replicating strategies used for pricing and hedging non-linear payoffs (options)
  - Used in many European insurance companies
Replicating Portfolio
How is it derived (high flying birds eye view)

Replicating universe (financial instruments) → Scenarios → Optimization → Insurance Assumptions → MCEV or future profit cash flows → Replicating portfolio

Cash flows from replicating instruments
Replicating Portfolio
How is it derived (low flying birds eye view)

• **Naive approach**
  Try to match discounted cash flows. Minimize:

\[ \sum_{Scenarios} \sum_{t=0}^{T} \left( \frac{CF(t, \text{Scenario}) - \sum_{i} A_i \cdot CF_i(t, \text{Scenario})}{\text{Num}(t, \text{Scenario})} \right)^2 \]

Widely used to replicate P&C Insurance liabilities.
Poor performance for Life & Pension insurance!
Because exact timing of the cash flows depends on management rules in the model

• **Approach that seems to work**
  Ignore the cash flow timing. Minimize:

\[ \sum_{Scenarios} \sum_{t=0}^{T} \left( \frac{CF(t, \text{Scenario}) - \sum_{i} A_i \cdot CF_i(t, \text{Scenario})}{\text{Num}(t, \text{Scenario})} \right)^2 \]
Replicating Portfolio
Why does it work

By the martingale property
\[ PVFP(t) = \text{Num}(t)E\left(PVFP(T)/\text{Num}(T)\right|\mathcal{F}_t) \]

The expression \( \sum_{t=0}^{T} CF(t, \text{Scenario})/\text{Num}(t, \text{Scenario}) \) is the value of \( PVFP(T)/\text{Num}(T) \) on a given Scenario.

Approximate the above sum by corresponding realisations for replicating instruments \( P_i(T)/\text{Num}(T) \).

Hope that \( E\left(PVFP(T)/\text{Num}(T)\right|\mathcal{F}_t) \approx E\left(\sum_i P_i(T)/\text{Num}(T)\right|\mathcal{F}_t) \) is a good approximation.

Note that the expected value on the right hand side can be computed analytically.
Replicating Portfolio
How is it validated

- Numerical stability of regression problem
- Conditional number
- Out-of-sample tests

- Avoid overfitting (offsetting long-short positions)
- Scatter plot and Q-Q plot

- Validity region
- Portfolio stability under small assumption changes
Replicating portfolio is a good approximation of the computed MCEV values

Optimization constraints don’t have a significant effect on the replicating portfolio

Using only basis scenarios in replication is not sufficient for a good fit of the stressed MCEV values
### Replicating Portfolio
How does it look like

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Bond 1Y</td>
<td>1,510,109,963</td>
</tr>
<tr>
<td>Zero Bond 3Y</td>
<td>-2,419,758,408</td>
</tr>
<tr>
<td>Zero Bond 5Y</td>
<td>770,205,262</td>
</tr>
<tr>
<td>Zero Bond 10Y</td>
<td>316,362,280</td>
</tr>
<tr>
<td>Zero Bond 15Y</td>
<td>-364,028,448</td>
</tr>
<tr>
<td>Zero Bond 20Y</td>
<td>693,283,763</td>
</tr>
<tr>
<td>Zero Bond 30Y</td>
<td>-258,964,464</td>
</tr>
<tr>
<td>Zero Bond 40Y</td>
<td>416,140,530</td>
</tr>
<tr>
<td>Forward Swap 1Y 10Y 3.5%</td>
<td>19,535,831</td>
</tr>
<tr>
<td>Forward Swap 5Y 10Y 3.5%</td>
<td>-170,604,480</td>
</tr>
<tr>
<td>Forward Swap 10Y 10Y 3.5%</td>
<td>-207,204,454</td>
</tr>
<tr>
<td>Forward Swap 15Y 10Y 3.5%</td>
<td>-360,334,649</td>
</tr>
<tr>
<td>Forward Swap 20Y 10Y 3.5%</td>
<td>-820,521,229</td>
</tr>
<tr>
<td>Forward Swap 30Y 10Y 3.5%</td>
<td>-115,412,326</td>
</tr>
<tr>
<td>Receiver Swaption 1Y 10Y 3.5%</td>
<td>-285,535,206</td>
</tr>
<tr>
<td>Receiver Swaption 5Y 10Y 3.5%</td>
<td>-1,023,153,975</td>
</tr>
<tr>
<td>Receiver Swaption 10Y 10Y 3.5%</td>
<td>-1,416,439,888</td>
</tr>
<tr>
<td>Receiver Swaption 15Y 10Y 3.5%</td>
<td>-1,284,008,480</td>
</tr>
<tr>
<td>Receiver Swaption 20Y 10Y 3.5%</td>
<td>-1,216,568,409</td>
</tr>
<tr>
<td>Receiver Swaption 30Y 10Y 3.5%</td>
<td>-1,901,652,495</td>
</tr>
</tbody>
</table>

- Replicating portfolio contains instrument that reflect the most significant risk factors
- Using replicating portfolios one can analyze MCEV distribution

Replicating portfolio distribution over 10000 Scenarios

99.5% Quantile
• Replicating portfolios are widely used in praxis to approximate the insurance liabilities

• Replicating portfolios perform very good if the liability profile is close to payoffs of replicating instruments

• Use of replicating portfolios in regulatory reporting framework requires a robust implementation and validation policy

• Replicating portfolios can be easily integrated in existing market risk management systems