

**FRIEDRICH-SCHILLER-
UNIVERSITÄT JENA**



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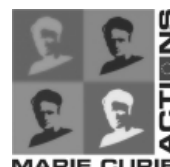
Spring School

**„Stochastic Models
in
Finance and Insurance”**

March 21 – April 1, 2011, Jena (Germany)

Programme

Sponsored by: Marie Curie Initial Training Network (ITN) “Deterministic and Stochastic Controlled Systems and Application” (PITN-GA-2008-213841), http://www.math.uaic.ro/~ITN_Marie_Curie/;
Friedrich-Schiller-University of Jena, <http://www.uni-jena.de>



Spring School "Finance and Insurance": First Week

TIME	MONDAY	TIME	TUESDAY	TIME	WEDNESDAY
08:45 – 9:00	Opening				
09:00 – 10:30	EBERLEIN	09:00 – 10:30	BÄUERLE	09:00 – 10:30	EBERLEIN
10:30 – 11:00	Coffee-Break	10:30 – 11:00	Coffee-Break	10:30 – 11:00	Coffee-Break
11:00 – 12:30	HÉNAFF	11:00 – 12:30	EBERLEIN	11:00 – 12:00	DENNSTÄDT
				12:00 – 12:30	KHOMENKO
12:30 – 14:00	LUNCH	12:30 – 14:00	LUNCH	12:30 – 13:00	Discussion
				13:00 – 14:30	LUNCH
14:00 – 15:00	Discussion Time	14:00 – 15:00	Discussion Time	14:30	GUIDED TOUR THROUGH JENA
15:00 – 15:45	EBERLEIN	15:00 – 15:45	REHMAN		
15:45 – 16:15	Coffee-Break	15:45 – 16:15	Coffee-Break		
16:15 – 17:00	EBERLEIN	16:15 – 16:45	GASSIAT		
17:00 – 17:30	JOOS	16:45 – 17:15	ANDRUSIV		
				18:00 –	ITN MEETING

Spring School "Finance and Insurance": First Week

TIME	THURSDAY	TIME	FRIDAY	SATURDAY	SUNDAY
09:000 – 10:30	CONT	09:000 – 10:30	VALKEILA	CONT	
10:30 – 11:00	Coffee-Break	10:30 – 11:00	Coffee-Break	Coffee-Break	
11:00 – 11:45	RAINER	11:00 – 12:30	CONT	VALKEILA	
11:45 – 12:30	SGARRA				
12:30 – 14:00	LUNCH	12:30 – 14:00	LUNCH	LUNCH	10:00 EXCURSION TO WEIMAR
14:00 – 15:00	Discussion Time	14:00 – 15:00	Discussion Time	WALK TO THE FOX TOWER	
15:00 – 15:45	CONT	15:00 – 15:45	KLÜPPELBERG		
15:45 – 16:15	Coffee-Break	15:45 – 16:15	Coffee-Break		
16:15 – 17:00	CONT	16:15 – 16:45	LIN		
17:00 – 17:30	BEDINI	16:45 – 17:15	JING		
		18:00 –		DINNER	

Springschool "Finance and Insurance": Second Week

TIME	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	TIME	FRIDAY
09:00 – 10:30	VALKEILA	LOISEL	SHEVCHENKO	BIAGINI	09:00 – 10:30	EDDAHBI
10:30 – 11:00	Coffee-Break	Coffee-Break	Coffee-Break	Coffee-Break	10:30 – 11:00	Coffee-Break
11:00 – 12:30	EL KAROUI	SHEVCHENKO	LOISEL	SHIRYAEV	11:00 – 12:30	SHIRYAEV
12:30 – 14:00	LUNCH	LUNCH	LUNCH	LUNCH	12:30 – 14:00	LUNCH
14:00 – 15:00	Discussion Time	Discussion Time	EXCURSION TO NAUMBURG	Discussion Time	14:00 – 14:45	KABANOV
15:00 – 15:45	MATICIUC	TIKANMÄKI		MEYER-BRANDIS	14:45 – 15:15	HAKASSOU
15:45 – 16:15	Coffee-Break	Coffee-Break		Coffee-Break	15:15	Coffee-Break and CLOSURE
16:15 – 16:45	OUAHHABI	YANG		JEANBLANC		
16:45 – 17:15		IBRAGIMOV				
18:00 –						

Spring School

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General Information

Arrival

Most participants will arrive on Sunday, March 20. Both railway stations 'Jena West' and 'Jena Paradies' are within walking distance to the hotels. If you have heavy luggage, it is most convenient to take a taxi.

Accommodation

Your accommodation will be booked at a hotel in the centre of Jena or close to it. If you arrive before 7 p.m., you don't have to take any further action and likewise if you are an invited lecturer. In case you arrive later than that you should contact the hotel staff to confirm your booking. Individual details will be sent to you by e-mail.

Lecture Rooms

The lectures will take place in the 'Rosensäle' (Rose Halls), Fürstengraben 27. This is right in the heart of town and close to the main building of the university. In front of and inside the building, signs will lead the way. The lecture room is equipped with a blackboard, overhead projectors, a notebook and a data projector.

Registration

The registration desk can be found in the 'Rosensäle' (Rose Halls), Fürstengraben 27. It will be open on Sunday, March 20, 5.00 - 8.00 p.m. and from Monday, March 21, to Wednesday, March 23, 8.00 - 9.00 a.m. and 12.30 - 1.30 p.m. For participants that arrive later we will open the registration desk on request. We provide name tags, a detailed programme and some additional material.

Welcome Reception

Participants arriving on Sunday, March 20, are invited to join the welcome reception which is scheduled for Sunday, March 20, 6.00 p.m.

Catering

In the morning breaks we will serve refreshments and coffee together with cookies and some fruit, and in the afternoon breaks also some cake. You can have lunch in the mensas at Ernst-Abbe-Platz and Philosophenweg or in the cafeteria in the university's main building. We have made arrangements so that you can enjoy rates, just show your name tag. Student card holders (of any university) are eligible for special student prices, cards must be shown. Everyone is expected to order and pay individually.

Guided Tour through Jena

On **Wednesday, March 23**, lectures will end in the early afternoon. At **2.00 p.m.** we will meet in front of the '**Rosensäle**' to take a guided tour through Jena. The language will be English, and there are no additional fees for you.

The history of the university and the town have always been intertwined and there are many sights worth to be visited.

Please let us know by registration until the **deadline Tuesday, March 22, 1.30 p.m.**, whether you are going to participate.

Afternoon Walk and Spring School Dinner

On **Saturday, March 26**, lectures will end at noon. If the weather permits, we will invite everyone who is interested to come along for a walk to the 'Fuchsturm' a few yards up the hill. There we will have coffee and enjoy a beautiful view over Jena. We will meet at **2.00 p.m.** in front of the '**Rosensäle**'.

In the evening we cordially invite all of you to the Spring School dinner, which will open at **6 p.m.** in the '**Rosensäle**', Fürstengraben 27. We will offer a rich buffet along with wine, beer and soft drinks. This is a good chance to get to know each other and to renew old relationships.

Please let us know by registration until the **deadline Tuesday, March 22, 1.30 p.m.** whether you intend to come to the dinner. It will be additional **15 € p.P.**

Spring School Excursion to Weimar

Approximately 20 kilometres west of Jena you can find the small university town Weimar, European Capital of Culture in the year 1999. During the recent centuries Weimar has been home of a great number of German poets and philosophers, for instance those of the Weimar Classicism (Ch.M. Wieland, J.W. Goethe, J.G. Herder, F. Schiller), of famous composers (J.S. Bach, F. Liszt, R. Wagner) and of artists and architects, in particular of those associated to the Bauhaus, which was founded here 90 years ago (W. Gropius, L. Feininger, P. Klee, W. Kandinsky). Also 90 years ago, Germany's first democratic constitution was adopted in the 'German National Theatre'. So the small town gave its name to the epoch of the Weimar Republic whose existence had ended with the ascent of the nazi regime. Traces of the 'Third Reich' can be found in the architecture of the town, but also in the remains of the former Buchenwald concentration camp close by.

We plan to take a local train from **station Jena West** to Weimar starting at **10.16 a.m. on Sunday, March 27**. There we will follow a guided tour. It will take around 2 hours, the language will be English. As the train connection is reliable and there is a train going from Weimar main station to Jena West every hour (about five minutes after the full hour), we will leave it up to you when you like to return to Jena. You can hardly make a mistake, but we will provide a few cell phone numbers of our local team to make sure you can contact someone if necessary.

The overall cost of the excursion (except lunch and other individual expenses) will be **10 € p.P.** Please let us know by registration until the **deadline Tuesday, March 22, 1.30 p.m.** whether you are going to participate.

Visit of the Naumburg Cathedral

About 1000 AD a castle named 'neweburg' was erected on the left bank of the river Saale, later it became 'Naumburg' today a small town with about 30.000 inhabitants a few kilometres north of Jena. Its most impressive building is the Cathedral St.

Peter and St. Paul, which grew out of a small parish church already mentioned in the chronicles of the year 1021. Its architecture is influenced by the Late-Romanic, but also by the Early- and Late-Gothic.

On **Wednesday, March 30**, we will meet at **1.45 p.m.** in front of the 'Rosensäle' and take a bus to Naumburg. We will visit the Cathedral and listen to a guide (in English). We will latest be back in Jena by 7 p.m.

This visit will be **10 € p.P.** Please let us know by registration until the **deadline Tuesday, March 22, 1.30 p.m.** whether you are going to participate.

Jena and the Friedrich-Schiller-University

When the elector Johann Friedrich I., who had supported the protestant reformation, had to flee from his former home Wittenberg, he settled down in Weimar and founded a 'high school' for clerics and teachers out of a former monastery in the small agricultural town of Jena at the river Saale. That was in 1548, and already a decade later, meanwhile a centre of the reformation, it was given the status of a university by the emperor.

About hundred years later scientific life blossomed in Jena. For instance, Erhard Weigel taught mathematics here, certainly his most famous student was G.W. Leibniz.

Later it was no other than Johann Wolfgang von Goethe who recruited many well known poets and philosophers of that time to teach in Jena. He also initiated the foundation of libraries, archives, the observatory and the botanical garden. Goethe closely cooperated with the chemist J.W. Döbereiner, who invented the earliest version of today's periodical system.

Hegel and Fichte worked in Jena and, of course, Friedrich Schiller, who taught history at the university though he is known to have used language on that job. His lectures were attended by Novalis, Hölderlin and Brentano.

Another great period for Jena and its university were the early days of industrialization. The physicist Ernst Abbe was still quite young when he issued a theory of microscopy, based upon years of practical experiences he had made together with the university mechanic Carl Zeiss. This successful line of research was continued by Otto Schott, who had got his PhD from the university. He founded the Zeiss laboratories to manufacture glasses and lenses in the highest quality. In his social ethics (even Ford could have been proud of) he was far ahead of his time. His company attracted many highly qualified workers, and that in turn made the town Jena prosper.

Also between the Wilhelminian period and the Weimar Republic famous scientists worked in Jena, among them the biologist Ernst Häckel, the mathematician and philosopher Gottlob Frege and the physicist Max Wien.

But already in the early days of national socialism the university of Jena turned into a role model. Many scientists aimed at theoretical foundations of racism and euthanasia, and victims of the nazi regime were abused in cruel medical experiments, even children. After World War II it took not very long until the university was again a playing field of ideology, this time the socialistic. The Friedrich-Schiller-University was one of the most important universities in the former GDR, also in mathematics. The most remarkable sign of the GDR period is the university tower. In the seventies and eighties Jena was a centre of subculture and dissidents, and several well known civil rights activists who were involved in the political change 1989 had studied in Jena.

Today the Friedrich-Schiller-University is the only full university of Thuringia, its number of students exceeds 20.000. Known to be a good place for intense and effective studies, it is ranked high in student polls. The university hosts an active research community, four DFG collaborative research centres and seven graduate schools. Particularly strong disciplines include the medical sciences, psychology, bioscience and physics. But also the faculty for mathematics and informatics enjoys a good national and international reputation.

The town Jena has about 100.000 inhabitants. Typical branches of industry and business are glass and optics, pharmacy, software, consulting and solar industry. Jena has a rich cultural life for instance regular concerts of the philharmonic orchestra or guest performances, modern and post modern theatre, many art galleries and exhibitions, and a lively scene of student and low-budget events. The town is surrounded by the beautiful landscape of the Saale valley flanked by forested hills and shell limestone rocks.

Spring School

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Abstracts

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Invited Lectures and Talks

Markov Decision Processes with Applications to Finance

Nicole Bäuerle

(KIT, Karlsruhe, Germany)

Markov Decision Processes are controlled Markov processes in discrete time. They appear in various fields of applications like e.g. economics, finance, operations research, engineering and biology. The aim is to maximize the expected (discounted) reward of the process over a given time horizon. We consider problems with arbitrary (Borel) state and action space with a finite and an infinite time horizon. Solution methods and the Bellman equation are discussed as well as the existence of optimal policies. For problems with infinite horizon we give convergence conditions and present solution algorithms like Howard's policy improvement or linear programming. The statements and results are illustrated by examples from finance and insurance like consumption-investment problems and dividend pay-out problems.

Evaluating Hybrid Products: the Interplay between Financial and Insurance Markets

Francesca Biagini

(Ludwig Maximilian University, Munich, Germany)

A current issue in the theory and practice of insurance and reinsurance markets is to find alternative ways of securitizing risks. To this purpose, insurance companies have tried to take advantage of the vast potential of capital markets by introducing exchange-traded insurance-linked instruments such as mortality derivatives and catastrophe insurance options. At the same time, insurance products such as unit-linked life insurance contracts, where the insurance benefits depend on the price of some specific traded stocks, offer a combination of traditional life insurance and financial investment. Furthermore, new kinds of insurance instruments, which insure against risks, connected to macro-economic factors such as unemployment, are recently offered on the market. In this lecture we will provide an introduction to how sophisticated mathematical methods for pricing and hedging financial claims can be applied to the valuation of the hybrid products mentioned above, as well as to premium determination, risk mitigation and claim reserve management.

Non-anticipative Functional Calculus: Theory and Applications

Rama Cont

(CNRS, Paris, France & Columbia University New York, USA)

We present a *non-anticipative functional calculus*, which extends the Ito calculus to path-dependent functionals of semimartingales in a non-anticipative way [1],[2],[3].

The approach builds on H. Föllmer's deterministic proof of the Ito formula [6] and a notion of pathwise functional derivative recently proposed by B. Dupire [5]. In this framework one can derive a functional extension of the Ito formula [1],[5], which has numerous applications in probability and mathematical finance.

The functional Ito formula is used to derive two key results. First, we obtain a martingale representation formula for square integrable functionals of a semimartingale S . Second, regular functionals S which have the local martingale property are characterized as solutions of a functional differential equation, for which a uniqueness result is given.

These results have various applications to the pricing and hedging of path-dependent contingent claims. We derive *universal* pricing and hedging equations which hold for any path-dependent option written on a financial asset whose price is modeled as a continuous semimartingale S . Using these results we derive a general formula for the hedging strategy of a path-dependent contingent claim and present a numerical method for computing this hedging strategy [4]. By contrast with methods based on Malliavin calculus, this representation is based on non-anticipative quantities which many be computed pathwise and leads to simple simulation-based estimators of hedge ratios [4].

OUTLINE

1. Motivation : sensitivity analysis of path-dependent options.
2. Functional representation of non-anticipative processes
3. Pathwise calculus for non-anticipative functionals
 - (a) Horizontal and vertical derivatives of a non-anticipative flow
 - (b) Functions of finite quadratic variation. Föllmer's pathwise Ito formula.
 - (c) A pathwise change of variable formula for non-anticipative flows
4. Functional Ito calculus
 - (a) Functional change of variable formula for semimartingales
 - (b) Vertical derivative of a non-anticipative process
 - (c) A martingale representation formula
5. Functional Ito calculus: extension to square integrable martingales
 - (a) Vertical derivative of a square integrable martingale
 - (b) General martingale representation formula
 - (c) Relation with Malliavin calculus
 - (d) Local martingales as solutions to functional Kolmogorov equations.
6. Applications to the pricing and hedging of path-dependent derivatives
 - (a) Hedging strategies for path dependent derivatives.
 - (b) Numerical computation of hedging strategies.
 - (c) A universal pricing equation for path dependent derivatives.

- (d) Theta-Gamma tradeoff for path-dependent options.
 - (e) Example: Asian options
7. Extensions
- (a) Functionals of quadratic variation.
 - (b) Example: weighted variance swaps.
 - (c) Locally regular functionals and functionals involving exit times.
 - (d) Examples: Barrier options.
8. Applications to stochastic control
- (a) Stochastic control problems and the martingale optimality principle.
 - (b) A verification theorem for non-Markovian stochastic control problems.
 - (c) Relation with Backward SDEs.

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Crucial Paths in Stochastic Modelling for Life Insurers

Nils Dennstedt

(Provinzial NordWest, Kiel, Germany)

Currently, stochastic modelling seems to be the right thing to do. European Insurance Supervisors (EIOPA) rely with Solvency II on stochastic modelling, insurers hence create models which incorporate simulation tools to make use of Monte Carlo Simulation methods.

Theoretic Approaches indicate the superiority over deterministic models and also over closed formulae. Thus, path dependency rules decision finding wherever you look. In fact, these approaches work perfectly on short term horizons in a well behaving financial market environment (deep and liquid, arbitrage free, and so forth) and with martingales risk neutral scenarios offer a fairly simple calculus.

This is about the point where mathematicians finalize their work. "Solutions exists", and now off to new horizons. But what happens now in the outside world, far away from scholastic desks and computer networks?

First off, finance experts take the approach and apply it with ease on pricing and accessing risk on exotic options, derivatives and complex structured products. Apparently, well behavior can be assumed on time horizons common for this type of problem. Afterwards, actuaries climb the stage after risk controllers claim stochastic modelling for any type of risk assessment. Do assumptions remain valid under extension of time spans? Now, models have to sufficiently cope with time horizons well beyond 35 years. Are markets deep and liquid? Do economic scenario generators work properly in this uncomfortable zone? What if curtosis plays a major role for the risk distribution? Are short term financial market instruments suitable to price a long term guarantee which is not even dealt OTC, where there might not even be a real market?

By the way, which risk measure is the best to choose? Is it value at risk or tail value at risk? Which are the most interesting questions to answer? The point of insolvency or the amount an insurer leaves for the industry to bear? Maybe possible buyers rather want to know how capital injections spread over quantiles rather than just look into the abyss in the tail.

Volatility remains a major concern as well. Will supervisors find an answer to volatile solvency quotas and will industry find a way to manage volatility in their capital assessments?

Some paths turn out to be crucial for life insurers. You are gladly invited to take a closer look.

Lévy Driven Financial Models

Ernst Eberlein

(University of Freiburg, Germany)

Empirical analysis of financial data reveals that standard diffusion models do not generate sufficiently accurate return distributions. To reduce model risk, more powerful classes of driving processes are appropriate. In this course exponential Lévy models and models driven by semimartingales in general are considered. Plain vanilla as well

as exotic options are priced in the new model class. As a further application in risk management we show how estimates of the risk of a portfolio of securities can be improved.

In the second part we develop a Lévy term structure theory. Three basic approaches are introduced: the forward rate model, the forward process model, and the LIBOR or market model. Pricing formulas for interest rate derivatives as well as efficient numerical algorithms to evaluate these formulas are derived. The LIBOR model is extended to a multi-currency and a credit setting. As an application pricing of cross-currency and a variety of credit derivatives is discussed.

A further topic to be discussed is a duality theory for models driven by Lévy processes. We conclude this survey by developing a new approach to quantify reserve capital in financial institutions.

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Chaos Expansion of some Functionals of the fBm and Lévy Processes and Applications

Mhamed Eddahbi

(Université Cadi Ayyad, Marrakesh, Morocco)

This talk is divided in three parts, in the first part we give the Wiener–Itô chaotic decomposition for the local time of the d -dimensional fractional Brownian motion with N -parameters and study its smoothness in the Sobolev–Watanabe spaces. As application, we derive some regularity properties of some additive functionals of the fractional Brownian motion that arise as limits in law of some occupation times of this process.

We close this part by studying the asymptotic behavior in Sobolev norm of the local time of the d -dimensional fractional Brownian motion with N -parameters when the space variable tends to zero, both for the fixed time case and when simultaneously time tends to infinity and space variable to zero.

In the second part we establish a Stroock formula in the setting of generalized chaos expansion for a certain class of Lévy processes, using a Malliavin type derivative based on the chaotic approach. As applications, we get the chaotic decomposition of the local time of a simple Lévy process as well as the chaotic expansion of the price of a financial asset and of the price of an European call option. We also study the behavior of the tracking error in the discrete delta neutral hedging under both the equivalent martingale measure and the historical probability.

The third part deals with existence and uniqueness results for Backward Stochastic Differential Equations driven by Teugel’s martingales and an independent Brownian motion. This kind of equations will be discussed under a locally Lipschitz conditions on the coefficient. The terminal data is assumed to be only square integrable. We prove that if the Lipschitz constant L_N behaves as $\sqrt{\log(N)}$ in the ball $B(0, N)$, then the corresponding BSDE has a unique solution. As application, we give a probabilistic interpretation for a large class of partial differential integral equations.

This talk is based on recent developments of the following references and the references therein.

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New Challenges between Finance and Insurance: the Longevity Risk A Microscopic Modelling Approach

El Karoui Nicole and Harry Bensusan
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Longevity risk is "the risk that members of some reference population might live longer on average than anticipated." It is a growing risk across the developed world as populations age, with a post-retirement life expectancy growing rapidly (by around 2 months a year for males and 1.5 months a year for females). Associated with this growth in an older population is a social need to develop products to allow individuals to secure lifetime income, and a business need to attract capital to support this new risk class. Traditionally, pension scheme actuaries used deterministic mortality tables, which only provide one estimate of mortality. However, to accurately quantify longevity risk, just using the traditional actuarial model is not sufficient, and stochastic models are necessary.

In recent years a variety of mortality models have been introduced, including the well known Lee and Carter (1992) model [5], widely used by insurance practitioners. They try to model and estimate the dynamics of the force of mortality for different ages x and dependency. The Cairns, Blake, and Dowd (CBD) model, [2] & [3], was developed to take into account more efficiently the cohort effect. However, these models fail to capture the so-called basic risk, due to the difference between the exposed population, (the pensioner or annuitant population) and the national population.

An alternative is the *microscopic modelling approach* (MMA), which can be used for populations where individuals are characterised not only by age, but also by additional indicators that are reflective of lifestyle and living conditions and may vary during the life of individual. As in a CBD models, environmental stochastic factors Y also influence the evolution. Inspired by works of Meleard, Tran, Ferriere, [6] we propose a stochastic individual-centered particle model in continuous time, where the individuals reproduce, age, interact and die. The dynamics is specified at the level of individuals. The model takes into account trait and age dependence of birth and death rates, where a 'trait' is a set of characters (genre, marital status, education, revenues) (d -dimensional vector). The MMA-model represents the discrete population at time t

by a point measure, whose the evolution mechanism is as follows. The birth rates and the death rates $b(t, Y, x, a)$, $d(t, Y, x, a)$ are dependig of the factors Y , the trait and the age of individuals. When a births occurs, the new individual has age 0 and characters determined bu his genitor. Also, individuals may be change of characters with some probability distribution also depending (t, Y, x, a) . Using Poisson Ponctuel measure, the model is easy to simulate. Simulation and asymptotic analysis provide information on the macro longevity risk, driven by a well-known non linear PDE equation. The calibration is done from the previous CDB model, using general data base, (in general national data base).

Combined with studies on demographic rates, such as fertility and immigration estimated on national population, microscopic modelling is applicable at social and political levels, offering guidance for strategic decision concerning, for example, immigration and retirement age policy.

Such models can provide useful benefits for the risk analysis of a given insurance portfolio, as shown by the study of benefits to be paid in the case of real portfolio of insured. An other application is the structuring of financial products allowing to insurers to transfer only their interest rate risk in financial markets.

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Measuring the Risk of Financial Models

Patrick Hénaff

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Conditional Default Probability and Density

Monique Jeanblanc

(University of Evry, France)

This paper proposes different methods to construct conditional survival processes, i.e, families of martingales decreasing with respect to a parameter. Conditional survival processes play a pivotal role in the density approach for default risk, introduced by El Karoui et al. Concrete examples will lead to the construction of dynamic copulae, in particular dynamic Gaussian copulae. It is shown that the change of probability measure methodology is a key tool for that construction. As in Kallianpur and Striebel, we apply this methodology in filtering theory to recover in a straightforward way, the classical results when the signal is a random variable.

No Arbitrage under small Transaction Costs

Yuri Kabanov

(Université de Franche-Comté, Besançon, France)

Replicating Life Insurance Liabilities

Oleksandr Khomenko

(ERGO Versicherungsgruppe AG, Düsseldorf, Germany)

Classical life insurance policies with profit participation are quite sophisticated financial products. On the one hand side, numerous plain vanilla and exotic securities in insurer's portfolio make the computation of the bonus amount rather complicated. On the other side, local regulation and taxation rules add further difficulties to the modeller's job. It seems that the only way to determine the value of the life insurance policy (or the value of the life insurance company) is by full Monte-Carlo simulation over the life time of a contract (at least 30-40 years). Due to large time horizon and extreme modelling complexity, a Monte-Carlo simulation with 1000 samples takes up to several hours to run.

If you want to access the risks of the life insurance policy or insurance company, you at least need to compute several stressed values corresponding to all relevant insurance

and market risks. Under Solvency II an estimation of a 99,5% quantile of the net asset value is required. Usually this is accomplished by another Monte-Carlo simulation with 10K samples. This makes the direct computation of risk figures (practically) impossible. Various approximation techniques have been developed for this purpose. Constructing replicating portfolios of life insurance liabilities is one of the most popular at the moment. In this talk we compare replicating portfolios to other known approaches and discuss various theoretical and practical aspects of replicating life insurance liabilities.

Modelling Electricity Market Data: The CARMA Spot Model, Forward Prices and the Market Risk Premium

Claudia Klüppelberg

(Technical University Munich, Germany)

We present a new model for the electricity market dynamics, which is able to capture seasonality, low-frequency dynamics and the extreme peaks in the spot price as well as the much less volatile forward prices. We introduce a non-stationary process for trend, seasonality and low-frequency dynamics, and model the large fluctuations by a non-Gaussian stable CARMA process.

We identify all components of our model, in particular, we separate the different components of our model and suggest a robust L_1 -filter to find the states of the CARMA process. We discuss possibilities for equivalent martingale measures in our heavy-tailed model, which leads to the estimation of the market price of risk and the risk premium in this market.

We apply this procedure to data from the German electricity exchange EEX. For this market we detect a clear negative risk premium, which indicates that the electricity producers are price takers willing to accept a lower price to hedge their production.

This is joint work with Fred Benth and Linda Vos from Oslo University.

Variable Annuities and Equity-linked Life Insurance

Stéphane Loisel

(ISFA, Université Lyon 1, France)

Computing Greeks without Derivatives

Thilo Meyer-Brandis

(Ludwig Maximilian University, Munich, Germany)

In Mathematical Finance the so-called "Greeks" are quantities that measure the sensitivity of an option price with respect to some model parameters involved. One of

the most prominent applications of Malliavin calculus in Mathematical Finance is the representation of these Greeks as expectation functionals that does not involve the derivative of the pay-off function of the option. Since most financial pay-off functions are not smooth this representation yields a numerically efficient way to compute Greeks. However, in the above mentioned representation of the Greeks the Itô diffusion that drives the price process of the underlying asset is assumed to have differentiable coefficients. For example, an extended Ornstein-Uhlenbeck process with switching mean reversion rate depending on the diffusion level, an important model in electricity price modelling, is not included in this class of Itô diffusions.

In this presentation we demonstrate how to generalize this application of Malliavin calculus to price processes driven by Itô diffusions with irregular drift coefficients. To this end, we study the general theoretical question of existence and Malliavin differentiability of strong solutions of stochastic differential equations (SDE's) with irregular drift coefficients. Using techniques from white noise analysis and a compactness criteria based on Malliavin calculus we develop a new method for the construction of strong solutions of SDE's with irregular coefficients. Further, this approach yields the additional important result that the constructed strong solutions are even Malliavin differentiable. This insight together with some "local time variational calculus" finally enables us to represent Greeks based on Itô diffusions with irregular drift coefficients as expectation functionals that neither involve the derivative of the pay-off function nor the derivatives of the diffusion coefficients.

The Risk Premium and the Esscher Transform in Power Markets

Carlo Sgarra

(Politecnico di Milano, Italy)

The purpose of the present paper is to investigate some relations between the risk premium and the change of measure in electricity markets. More precisely we shall focus our attention on a peculiar feature of the risk premium, the sign change. In power markets, the usual pricing approach based on the construction of an equivalent martingale measure is not viable any more. Electricity, for example, is a non-storable commodity, so it does not make neither sense to trade in the underlying nor to use hedging arguments. However, since the forward contracts need to have a price dynamics being arbitrage-free, these prices can still be considered to be discounted expectations of the final value of the underlying, but with respect to any equivalent probability measure. Alternatively, it is possible to think about forward prices in terms of risk premium: this is defined as the difference between the forward prices computed with respect to the risk-neutral measure and with respect to the objective measure, respectively (see Geman [19]). Once the probability measure with respect to which the discounted prices must be calculated has been chosen, then the risk premium is defined in a unique way. The peculiarities of the risk premium in energy markets have been thoroughly investigated during the past few years and many evidences have been collected on its behavior. The theory of normal backwardation suggests that producers of a commodity wish to hedge their revenues by selling forwards, so they accept a discount on the expected spot

price. Thus, we should have the forward prices of a commodity less than the expected value with respect to the objective probability measure. So, in this context, the risk premium should be negative. On the other hand, several authors found evidence of a positive risk premium for contracts with a short time to maturity: Geman and Vasicek [20] investigated the Pennsylvania-New Jersey-Maryland electricity market and justified the existence of a positive risk premium by the market's aversion for the high volatility and consequently willingness to pay high prices to ensure delivery. In the same study, for contracts with longer maturities, the sign of the risk premium changes. Longstaff and Wang [26] perform a non-parametric study of the same market obtaining evidence of a positive risk premium for the short-term contracts. Their study has been extended by Diko, Lawford and Limpens, who analyse risk premia in the three markets EEX, Powernext and Dutch market APX [14]. A term structure for the risk premium is found there, which varies significantly from the short-term maturities to long-term maturities. A recent study with a systematic investigation on the short-term maturity Nord Pool market has been provided by Lucia and Torrò [28] who extend a previous study by Botterud, Bhattacharya and Ilic [9]: they find evidence of time-varying risk premium and of its positivity for short maturities. Benth, Carlea and Kiesel [3] provide an interpretation of the risk premium in electricity market in terms of the certainty equivalent principle and jumps in the spot price dynamics, whereas Benth and Meyer-Brandis [6] provide an information-based approach for explaining the risk premium.

In this paper we are going to provide the mathematical evidence of the risk premium sign change on the basis of the most popular models available in the literature and of the most natural probability measure change: the Esscher transform. This measure change, introduced by Esscher [17] in an actuarial context, has been extensively applied in derivative pricing since the pioneering work by Gerber and Shiu [21], who extended the original idea by Esscher to a Lévy framework. It has been further extended to semimartingale modelling by Kallsen and Shiryaev [24], and its popularity in the Lévy framework is due to two very important reasons: the first is that it enjoys some relevant optimality properties; its close relationship with the minimal entropy martingale measure has been thoroughly investigated in the papers by Esche and Schweizer [16] and by Hubalek and Sgarra [22]; the second, the most important for the present purposes, is that it preserves the independent increment property. This important feature of the Esscher transform, which has been proved for the linear Esscher martingale transform for Lévy processes, still holds when the increments are not stationary any more. This is the main reason to justify our choice of the Esscher Transform as a reference measure change: if the structure of the spot dynamics it is not preserved by the measure change, it is almost impossible to obtain relevant information on the forward prices and in particular any explicit formulas for evaluation. In the framework of spot price dynamics described by independent increment processes we shall show that a measure change performed via the Esscher transform can justify the sign change of the risk premium between short and longer maturities.

We obtain general results on the application of the Esscher transform to power markets. Our spot model cover many of the important cases applied in practise and theory, and we show that the choice of pricing measure provided by the Esscher transform yields analytically tractable models for forward pricing. We treat both geometric and arithmetic models, the latter is also sufficiently flexible for allowing for pricing of power forwards which deliver the underlying over a period. Moreover, it is proven that the Esscher transform corresponds to a change in the mean-reversion level for

the spot, much in line with the classical Girsanov transform with a constant drift. In this respect, the Esscher transform is a true generalization of the Girsanov transform to processes with independent increments. The constant Girsanov change of measure seems to be the standard choice in the power markets. The explicit forward prices obviously imply analytical expressions for the risk premia. We analyse the particular case of a two-factor spot model, with the first factor modelling the base component of the prices, while the second is the spike component. Spikes are large sudden price increases being fastly reverted back. We discuss the consequences of different choices of the market price of risk, being the parameters in the Esscher transform. As it turns out, spikes occurring seasonally may explain the occurrence of a positive risk premium in the short end, while in the long end we can have a negative premium. If we are close to a period with high spike intensity, the premium may become positive, while if the spiky period is far in the future, we see a risk premium being negative, or backward-dated. However, spike periods are related to bumps in the risk premium curve. Taking into account that the premium is scaled by the current states of the factors driving the spot price, we find that the risk premium can vary stochastically, and that it can have periods of a positive premium in the short end and negative in the long. In fact, the Esscher transform provides a large degree in flexibility even for constant market prices of risk.

Our results are presented as follows. Section 2 will recall the basic modelling framework we are going to assume: we shall introduce two classes of models based on independent increment processes: the geometric and the arithmetic models. In the third section we shall resume the essential features of the Esscher transform for independent increment processes. Furthermore, we present the forward prices obtained in both the geometric and the arithmetic models. Evidence of the risk premium sign change is shown in the main Section 4, where we also provide models explaining the change of risk premium as coming from seasonally varying spike intensities. We consider also forwards delivering over a period (so-called flow forwards), and the question of the Esscher transform being a martingale measure. Finally, in section 7 we conclude and outline some futures perspectives of the present work.

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Fractional and Multifractional Models in Finance

Georgiy Shevchenko

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Brownian motion, or Wiener process, had been a popular tool in the last century for modeling randomness in natural and sciences, computer networks and financial markets. However, the lack of memory in this model led many scientists to consider other models, like fractional Brownian motion, which allow for a long memory or/and a long-range dependence.

Nowadays, the new dimensions created by the climate change and the global financial crisis force us to consider more complex models having properties of multifractionality, which means that the deepness of memory may depend on the time instance, the state of a system and/or the time scale.

In my talk I plan to discuss the place of fractional and multifractional models in finance. I will give an overview of the current state-of-art for these models, describing both financial and mathematical properties of them.

Standard and Nonstandard Problems in Optimal Stopping

Albert N. Shiryaev

(*MSU and Steklov Institute Moscow, Russia*)

The classical optimal stopping problem for a Markov process $X = (X(t), (F(t)), P^x)$, $t \leq T$, consists of finding the value function $V(x) = \sup E^x G(\tau, X(\tau))$ where sup is taken on $(F(t))$ -stopping times τ . In our lectures we intend to consider several nonstandard optimal stopping problems where instead of the gain function $G(\tau, X(\tau))$ we have functions $G(\tau, X(\tau); \theta)$, where $\theta = \theta(\omega)$ is a random variable not necessarily $F(\tau)$ -mesurable (for example where θ is time where process $X = X(t)$, $t \leq T$, reaches its maximal value, $B(\theta) = \max B(t)$, max is taken on all $t \leq T$).

Topics on Fractional Brownian Motion

Esko Valkeila

(*Aalto University School of Science, Finland*)

1. Basic properties of fBm
2. Lebesgue-Stieltjes integrals
3. Transformations
4. Characterization of fBm
5. Change of variables formulas for fBm
6. Arbitrage with fBm
7. Mixed models

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Contributed Talks

On Minimal Entropy Martingale Measures

Andrii Andrusiv

(Friedrich Schiller University of Jena, Germany)

In this talk we consider incomplete financial markets in which the price process of the risky asset follows an exponential compound Poisson process. Our attention is focused on the minimal entropy martingale measure (MEM) and the investigation of the relation between the MEM and the Esscher martingale measure.

Information and Credit Risk

Matteo Bedini

(Friedrich Schiller University of Jena, Germany)

The object of this work is a model for the information concerning the default of a financial firm. The definition of the "information process" is the starting point of a new class of models for credit-risk. The information process aims to model the qualitative behaviour of the information concerning a default event. The main mathematical properties of this approach are discussed in a first step. Some theoretical aspects related with the theory of enlargement of filtrations are then considered in the second part.

Investment Consumption Problem in Illiquid Markets with Regime Switching

Paul Gassiat

(LPMA, Université Paris 7, France)

We study the problem of optimal investment/consumption over infinite horizon in an illiquid market, where the investor is restricted to trading at random times. The asset price and trading times intensity follow regime-switching dynamics representing different liquidity cycles in the market. We give a characterization of the value function for this problem in terms of viscosity solution for a system of IPDEs. In the case of power utility, we prove smoothness results and the existence of optimal strategies characterized in feedback form. Finally, we show the convergence of an iterative method to compute the value function and present some numerical results illustrating the impact of our liquidity constraints.

Based on joint work with F. Gozzi (Luiss university, Roma) and H. Pham (University Paris Diderot).

A Study of a Stochastic Differential Equation with Super Linear Growth Rates

Antoine Hakassou

(Université Cadi Ayyad, Marrakesh, Morocco)

We consider the following stochastic differential equation:

$$(1) \quad X_t = x + \int_0^t X_s \log |X_s| ds + \int_0^t X_s \sqrt{|\log |X_s||} dW_s$$

where $(W_t)_{t \geq 0}$ is a standard Brownian motion.

It is well known that the s.d.e (1) admits a weak solution to a lifetime $\zeta \equiv \infty$. Moreover the results of Yamada and Watanabe shows that if s.d.e (1) admits the path-wise uniqueness property, then it admits a strong solution. Thus, the study of path-wise uniqueness is of great interest. In this work, we seek the pathwise uniqueness, the non explosion and the non confluence of solution.

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Parabolic SDE in Infinite Dimensions over G-expectatins

Anton Ibragimov

(Università di Milano-Bicocca, Milano, Italy)

We consider ordinary probability space and set of H-valued random variables, where H is separable Hilbert space. We introduce there sublinear expectation functional (instead of classical expectation) under which we can construct normal distributed r.v. (G-normal distribution), Brownian motion (G-brownian motion), another special processes in H and so on. We can relate G-Brownian motion to a fully nonlinear partial differential equation that the viscosity solution of its equation is expressed by that G-Brownian motion. After introducing stochastic integral over G-expectation we can involve in the same way OrnsteinUhlenbeck process for finding viscosity solution of general type parabolic PDE with first order derivative term. So, in this talk we try to give an explicit notion of viscosity solutions of some types of parabolic PDEs in infinite

dimension case and their connection to theory of G-expectations.

SPDE driven by a Fractional Brownian Motion of Hurst Coefficient $H \in (1/2, 1)$. Study through its Doubly Stochastic Interpretation

Shuai Jing

(Université de Bretagne Occidentale, Brest, France)

We first state a special type of Itô formula involving stochastic integrals of both a standard and a fractional Brownian motion. Then we apply Doss-Sussman transformation to establish the link between backward doubly stochastic differential equations, driven by both standard and fractional Brownian motions, and backward stochastic differential equations, driven only by standard Brownian motions. Following the same technique, we further study associated nonlinear stochastic partial differential equations driven by fractional Brownian motions and partial differential equations with stochastic coefficients.

Modelling the Development of Interbank Markets during a Liquidity Shock

Dominik Joos

(KIT, Karlsruhe, Germany)

We study the problem of maximizing expected utility of terminal wealth in a two-period market with risky and non-risky assets for a continuum of banks. Following a liquidity shock on individual bank level, we allow for secured and unsecured interbank markets to develop. Prices are set by a Walrasian actioneer. We determine the banks' optimal behaviour in a general setting using KarushKuhnTucker conditions and derive explicit optimal strategies in special cases. As a result, the existence of interbank markets smoothes out the liquidity shocks. The spread between interest rates can be calculated with a formula depending on the distribution of the shock.

Nash Equilibria for 2-Persons Non-Zero Sum Stochastic Differential Games in a general Setting

Qian Lin

(Université de Bretagne Occidentale, Brest, France)

In this talk we study Nash equilibrium payoffs for nonzero-sum stochastic differential games via the theory of backward stochastic differential equations. We obtain

an existence theorem and a characterization theorem of Nash equilibrium payoffs for nonzero-sum stochastic differential games with nonlinear cost functionals defined with the help of a doubly controlled backward stochastic differential equation. Our results extend former ones by Buckdahn, Cardaliaguet and Rainer (2004) and are based on a backward stochastic differential equation approach.

Multivalued Backward Stochastic Differential Equations driven by Fractional Brownian Motion with Hurst Parameter $H > 1/2$

Maticiuc

(*University "Al. I. Cuza", Iași, Romania*)

Generalized Real Harmonisable Multifractional Stable Process and its Path Properties

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The real harmonizable multifractional stable process (rhmsp for short) was introduced by Douzi and Shevchenko (2010), which has both properties of heavy tails and multifractionality, it can be regarded both as multifractional generalization of a harmonizable fractional stable process with Hurst parameter H and a stable generalization of harmonizable multifractional Brownian motion.

The rhmsp can be defined by replacing the Hurst parameter H by a Hölder function $H(t)$.

Our main interest in this talk is to introduce a new process which can be a generalization of rhmsp, which will be called a Generalized real harmonizable multifractional stable process. This process will also depend on a functional parameter $H(t)$ that belongs to a set \mathcal{H} , but \mathcal{H} will be much more larger than the space of Hölder functions.

We also study its path properties: continuity, the localizability and the existence of local time.

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Hölder Regularity for Viscosity Solutions of fully Nonlinear Hamilton-Jacobi Equations with Super-Quadratic Growth in the Gradient

Catherine Rainer

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Viscosity solutions of fully nonlinear, local or non local Hamilton-Jacobi equations with a super-quadratic growth in the gradient variable are proved to be Hölder continuous, with a modulus depending only on the growth of the Hamiltonian. The proof involves some representation formula for nonlocal Hamilton-Jacobi equations in terms of controlled jump processes and a weak reverse inequality.

Continuity Estimates of the Optimal Exercise Boundary with respect to Volatility for the American Option

Nasir Rehman

(AIOU University, Islamabad, Pakistan)

In this talk we consider the Garman-Kohlhagen model for the American foreign exchange put option in one-dimensional diffusion model where the volatility and the domestic and foreign currency risk-free interest rates are constant. First we make preliminary estimate regarding the optimal exercise boundary and then the continuity estimate with respect to volatility for the value functions of the corresponding options. Finally we establish the continuity estimate for the optimal exercise boundary of the American foreign exchange put option with respect to the volatility parameter.

How to Hedge Asian Options in Fractional Black-Scholes Model

Heikki Tikanmäki

(Aalto University School of Science, Finland)

We prove change of variables formulas [Itô formulas] for both arithmetic and geometric averages of geometric fractional Brownian motion. They are valid for all convex functions, not only for smooth ones. Moreover, they can be used for obtaining hedges (but not prices) for Asian options in fractional Black-Scholes model. We get explicit hedges in some cases where explicit hedges are not known even in the ordinary Black-Scholes model.

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Existence and Uniqueness of Invariant Measure of SPDE with Reflection

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