Lower and upper bounds of martingale measure densities in continuous time markets

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Workshop: Finance and Insurance

presentation based on a joint work with Inga B. Eide
Outlines

1. Market modeling: EMM and no-arbitrage pricing principle
2. Framework: Claims and price operators
3. No-arbitrage pricing and representation theorems
4. EMM and extension theorems for operators
5. A version of the fundamental theorem of asset pricing

References
Market modeling is based on a probability space \((\Omega, \mathcal{F}, P)\) identifying the possible future scenarios. The probability measure \(P\) is derived from DATA and/or EXPERTS’ BELIEFS of the possible scenarios and the possible dynamics of the random phenomenon.

On the other hand the modeling of asset pricing is connected with the idealization of a FAIR MARKET. This is based on the principle of no-arbitrage and its relation with a risk neutral probability measure \(P^0\), under which all discounted prices are (local) martingales with respect to the evolution of the market events (for this reason \(P^0\) is also called martingale measure). The probability space \((\Omega, \mathcal{F}, P^0)\) provides an efficient mathematical framework.
Fundamental theorem of asset pricing

Naturally, we would like the models of mathematical finance to be both consistent with data analysis and to be mathematically feasible.

This topic was largely investigated for quite a long time yielding the various versions of the *fundamental theorem of asset pricing*. The basic statement is:

For a given market model on \((\Omega, \mathcal{F}, P)\) and the flow of market events \(\mathcal{F} = \{\mathcal{F}_t \subseteq \mathcal{F}, t \in [0, T]\}\) \((T > 0)\), satisfying some assumptions, there exists a martingale measure \(P^0\) such that \(P^0 \sim P\), i.e.

\[
P^0(A) = 0 \iff P(A) = 0, \quad A \in \mathcal{F}_T.
\]

Cf. e.g. Delbaen, Harrison, Kreps, Pliska, Schachermayer.
No-arbitrage pricing principle

Mathematically the existence of an *equivalent martingale measure* (EMM) implies the absence of *arbitrage opportunities*, this embodying the economical fact that in a “fair” market there should be no possibility of earning riskless profit.

In fact the *principle of no-arbitrage* provides the basic pricing rule in mathematical finance:

*For any claim* $X$, achievable at time $t$ and purchased at time $s$, *its “fair” price* $x_{st}(X)$ *is given by*

$$x_{st}(X) = E_0\left[\frac{R_s}{R_t}X|\mathcal{F}_s\right].$$

*Here $R_t$, $t \in [0, T]$, represents some riskless investment always achievable and always available on the market (the numéraire).*
The martingale measure $P^0$ used in the no-arbitrage evaluation is not necessarily unique: the equivalent martingale measure is unique if and only if the market is complete, i.e. if all claims are attainable in the market.

In an incomplete market, if the claim $X$ is attainable, the no-arbitrage evaluation of the price is independent of the choice of the martingale measure applied, thus the price is unique.

But, if $X$ is not-attainable, then the no-arbitrage principle does not give a unique price, but a whole range of prices that are equally valid from the no-arbitrage point of view.

Many authors have been engaged in the study of how to select a martingale measure to be used. The approaches have been different.
Selection of one measure that either is in some sense optimal or whose use is justified by specific arguments in incomplete markets. Without aim or possibility to be complete we mention the minimal martingale measure and variance-optimal martingale measure which are both in some sense minimizing the distance to the physical measure (ref. e.g. Schweizer 2001). The Esscher measure is motivated by utility arguments to justify its use and it is also proved that it is also structure-preserving when applied to Lévy driven models (ref. e.g. Delbaen et al. 1989, Gerber et al. 1996).

Instead of searching for the unique "optimal" equivalent martingale measure, one can try to characterize probability measures that are in some sense "reasonable". This is the case of restrictions of the set of EMM in such a way that not only arbitrage opportunities are ruled out, but also deals that are "too good" as in the case of bounds on the Sharpe ratio (the ratio of the risk premium to the volatility). Ref. e.g. Cochrane et al. (2000), Björk et al. (2006), Staum (2006).
In various stochastic phenomena the impact of some events is crucial both for its own being and for the market effects triggered. As example, think of the devastating effects of natural catastrophies (e.g. earthquakes, hurricanes, etc.) and epidemies (e.g. SARS, bird flu, etc.).

These events, though devastating, occur with small, but still positive probabilities.

Having this in mind, the choice of a ”reasonable” EMM should take in to account a proper evaluation of ”small probabilities”, i.e.

” $P(A)$ small” $\iff$ ” $P^0(A)$ small.”

Note in fact that the assessment under $P$ of the risk of these events incurring can be seriously misjugded under a $P^0$ only equivalent to $P$. This is particularly relevant for the evaluation of insurance linked securities.
Goal

We study the existence of EMM $P^0$ with densities $\frac{dP^0}{dP}$ lying within pre-considered lower and upper bounds:

$$0 < m \leq \frac{dP^0}{dP} \leq M < \infty \quad P\text{-a.s.}$$

We have to stress that these bounds are random variables:

$$m = m(\omega), \quad \omega \in \Omega,$$
$$M = M(\omega), \quad \omega \in \Omega.$$
A bit on related literature:

- In Roklin and Schachermayer (2006), we follow the study on the existence of a martingale measure with lower bounded density.

- In Kabanov and Stricker (2001) (see also Ràzonyi (2002)) densities are bounded from above. Here the study shows that the set of equivalent $\sigma$-martingale measures with density in $L_\infty(F)$ is dense (in total variation) in the set of equivalent $\sigma$-martingale measures.

Here, we consider lower and upper bounds for martingale measure densities simultaneously.
2. Framework

We consider a continuous time market model without friction for the time interval $[0, T]$ ($T > 0$) on the complete probability space $(\Omega, \mathcal{F}, P)$. The flow of information is described by the filtration

$$\mathbb{F} := \{ \mathcal{F}_t \subseteq \mathcal{F}, t \in [0, T] \}$$

augmented of the $P$-zero events, right-continuous and such that $\mathcal{F} = \mathcal{F}_T$.

Claims.

For any fixed $t$, the achievable claims in the market payable at $t$ constitute the convex sub-cone

$$L_t^+ \subseteq L_p^+(\mathcal{F}_t)$$

of the cone

$$L_p^+(\mathcal{F}_t) := \{ X \in L_p(\Omega, \mathcal{F}_t, P) : X \geq 0 \}, \quad p \in [1, \infty).$$
N.B. The case
\[ L_t^+ = L_p^+(\mathcal{F}_t) \quad \text{for all } t \in [0, T] \]
corresponds to a complete market. Otherwise the market is incomplete, i.e.
\[ L_t^+ \subset L_p^+(\mathcal{F}_t) \quad \text{for at least a } t \in [0, T]. \]
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N.B. If borrowing and short-selling is admitted, then the variety of claims is a linear sub-space

\[ L_t \subseteq L_p(\mathcal{F}_t) : \quad L_t := L_t^+ - L_t^+, \]

i.e. \( X \in L_t \) can be represented as \( X = X' - X'' \) with \( X', X'' \in L_t^+ \).
Price operators

If $X \in L_t^+$ is available at time $s: s \leq t$, then its price $\hat{x}_{st}(X)$ is an $\mathcal{F}_s$-measurable random variable such that

$$\hat{x}_{st}(X) < \infty \quad P - a.s.$$
Price operators

If $X \in L_t^+$ is available at time $s : s \leq t$, then its price $\hat{x}_{st}(X)$ is an $\mathcal{F}_s$-measurable random variable such that

$$\hat{x}_{st}(X) < \infty \quad P-a.s.$$ 

**N.B.** If short-selling is admitted, the price operators are defined on the sub-space $L_t \subseteq L_p(\mathcal{F}_t)$ according to

$$\hat{x}_{st}(X) := \hat{x}_{st}(X') - \hat{x}_{st}(X'')$$

for any

$$X \in L_t : \quad X = X' - X'' \quad \text{with} \quad X', X'' \in L_t^+.$$
Any $s, t : s \leq t$, the price operator $\hat{x}_{st}(X), X \in L^+_t$, satisfies:

- It is **strictly monotone**, i.e. for any $X', X'' \in L^+_t$ available at $s$
  
  \[
  \hat{x}_{st}(X') \geq \hat{x}_{st}(X''), \quad X' \geq X'',
  \]
  
  \[
  \hat{x}_{st}(X') > \hat{x}_{st}(X''), \quad X' > X''
  \]

  The notation “$\geq$” represents the standard point-wise “$\geq P\text{-a.s.}$”, while “$>$” means that, in addition to “$\geq P\text{-a.s.}$”, the point-wise relation $>$ holds on an event of positive $P$-measure.

- It is **additive**, i.e. for any $X', X'' \in L^+_t$ available at $s$
  
  \[
  \hat{x}_{st}(X' + X'') = \hat{x}_{st}(X') + \hat{x}_{st}(X'').
  \]

- It is $\mathcal{F}_s$-**homogeneous**, i.e. for any $X \in L^+_t$ available at $s$ and any $\mathcal{F}_s$-multiplier $\lambda$ such that $\lambda X \in L^+_t$, then
  
  \[
  \hat{x}_{st}(\lambda X) = \lambda \hat{x}_{st}(X).
  \]

We set $\hat{x}_{tt}(X) = X, X \in t^+$. 
Definition. The price operator \( \hat{x}_{st}(X), X \in L_t^+, \) is tame if

\[
\hat{x}_{st}(X) \in L_p(\mathcal{F}_s), \quad X \in L_t^+,
\]

i.e. \( \|\hat{x}_{st}(X)\|_p := \left( E(\hat{x}_{st}(X))^p \right)^{1/p} < \infty. \)

N.B. This definition is motivated by forthcoming arguments on time-consistency of the price operators.

In view of the forthcoming no-arbitrage arguments we consider only tame price operators.
Discounting

To be able to compare prices over time, we consider a numéraire which represents the chosen “unit of measurement” for money. This is an asset always available with payoff $R_t > 0$ $P$-a.s. for every $t$ and

$$R_s = \hat{x}_{st}(R_t), \quad s \leq t,$$

$$\| R_{s'} - R_s \|_p \longrightarrow 0, \quad s' \downarrow s.$$

We set $R_0 = 1$. Hence it is natural to assume that, for any $X \in L_t^+$

$$\frac{X}{R_s} \in L_t^+, \quad s \leq t$$

and we also assume that for every $s \leq t$ there exist $c_{st}, C_{st} > 0$ constants such that

$$c_{st} \leq \frac{R_t}{R_s} \leq C_{st}.$$
N.B. We have that

\[ 1 = \frac{\hat{x}_{st}(R_t)}{R_s} = \hat{x}_{st}\left(\frac{R_t}{R_s}\right) \in L_s^+. \]

Thus \(1 \in L_t^+\) for all \(t \in [0, T]\).

**Definition.** The *discounted price operator* is defined as:

\[ x_{st}(X) := \frac{\hat{x}_{st}(X)}{R_s} = \hat{x}_{st}\left(\frac{X}{R_s}\right), \quad s \leq t. \]

Naturally, \(x_{st}(R_t) = 1, \ s \leq t.\)

N.B. The discounted price operator \(x_{st}\) inherits the properties of strict monotonicity, additivity and \(\mathcal{F}_s\)-homogeneity from \(\hat{x}_{st}\). Thus it is itself a price operator.
Moreover, the discounted price operator is tame if and only if the price operator is tame.
Here we consider the family of price operators of $X \in L_t^+, \ t \leq T$,

\[ \hat{x}_{st}(X), \ 0 \leq s \leq t, \]
\[ x_{st}(X), \ 0 \leq s \leq t. \]

**Definition.** The family of the prices above is *right-continuous* at $s$ if $X$ is available for some interval of time $[s, s + \delta]$ ($\delta > 0$) and

\[ \|\hat{x}_{s't}(X) - \hat{x}_{st}(X)\|_p \rightarrow 0, \quad s' \downarrow s. \]

**N.B.** The family of discounted prices is right-continuous if and only if the family of the original prices is.

**Definition.** Let $\mathcal{T} \subseteq [0, T]$. The family $x_{st}, \ s, t \in \mathcal{T} : s \leq t$, of tame discounted price operators $x_{st}(X), X \in L_t^+$, is *time-consistent* (in $\mathcal{T}$) if for all $s, u, t \in \mathcal{T}: s \leq u \leq t$

\[ x_{st}(X) = x_{su}(x_{ut}(X)), \]

for all $X \in L_t^+$ such that $x_{ut}(X) \in L_u^+$. 

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Comment. This axiomatic approach to price processes is inspired by risk measure theory. The requirements of monotonicity, additivity, and homogeneity are related to the concept of coherent risk measures. The additional assumption of strict monotonicity, is related to the concept of a relevant risk measure.
3. No-arbitrage pricing and representation theorems

We can reformulate the basic statement of the fundamental theorem saying that: *The absence of arbitrage is ensured by the existence of an EMM $P^0$, such that the discounted prices $x_{st}(X)$, $X \in L^+_t$, admit the representation*

$$x_{st}(X) = E^0[X|\mathcal{F}_s], \quad X \in L^+_t.$$  

Moreover, for any $t \in [0, T]$ and $X \in L^+_t$ the *price process*

$$x_{st}(X), \enspace 0 \leq s \leq t$$

is a *martingale* with respect to the measure $P^0$ and the filtration $\mathbb{F}$.

**Definition.** A probability measure $Q \sim P$ is *tame* if, for every $t \in [0, T]$, we have that

$$E_Q[X|\mathcal{F}_t] \in L_p(\mathcal{F}_t), \quad X \in L_p(\mathcal{F}_T).$$
Facts. Let us consider $P^0 \sim P$ and tame probability measure. The study of the operator conditional expectation:

$$E^0[\cdot | \mathcal{F}_s] : L_p(\mathcal{F}_t) \longrightarrow L_p(\mathcal{F}_s)$$

shows that it is: tame, strictly monotone, linear, and $\mathcal{F}_s$-homogeneous. Hence it has all the properties of a tame price operator on the whole $L_p^+(\mathcal{F}_s)$.

Moreover, the family of conditional expectations is time-consistent:

$$E^0[X|\mathcal{F}_s] = E^0[E^0[X|\mathcal{F}_u]|\mathcal{F}_s], \quad X \in L_p(\mathcal{F}_t), \quad 0 \leq s \leq u \leq t,$$

and also right-continuous.

Quite remarkably, it turns out that the converse is also true: any tame price operator $x_{st}(X), \ X \in L_p(\mathcal{F}_t)$, admits representation as conditional expectation with respect to an equivalent martingale measure.
Lemma. For \( s, t \in [0, T] : s \leq t \) fixed, the operator \( x_{st}(X), X \in L_p(\mathcal{F}_t) \), is monotone linear \( \mathcal{F}_s \)-homogeneous if and only if it admits representation

\[
(1) \quad x_{st}(X) = E^0_{st}[X|\mathcal{F}_s], \quad X \in L_p(\mathcal{F}_t),
\]

with respect to a probability measure

\[
P^0_{st}(A) = \int_A f_{st}(\omega) P(d\omega), \quad A \in \mathcal{F}_t,
\]

where \( f_{st} \in L^+_q(\mathcal{F}_t), \frac{1}{q} + \frac{1}{p} = 1 \). Moreover the operator is strictly monotone if and only if \( f_{st} > 0 \) \( P \)-a.s. In addition, the operator (1) is bounded (continuous) if and only if

\[
\begin{cases}
\text{ess sup } E[f_{st}^p|\mathcal{F}_s] < \infty, & p \in (1, \infty) \\
\text{ess sup } f_{st} < \infty, & p = 1
\end{cases}
\]

and is tame if and only if \( P^0_{st} \) is tame.
The results above are restricted to the two fixed time points \( s \leq t \). Now we keep \( s \) fixed and we compare the representations for different time points \( u \leq t \).

**Theorem.** Let \( s, t \in [0, T] : s \leq t \). Assume that the operators

\[
x_{su}(X), \quad X \in L_p(\mathcal{F}_u), \quad s \leq u \leq t,
\]

are tame price operators constituting a time-consistent family. Then, for all \( u \in [s, t] \), the representation

\[
(2) \quad x_{su}(X) = E_{st}^0[X|\mathcal{F}_s], \quad X \in L_p(\mathcal{F}_u),
\]

holds in terms of the measure \( P_{st}^0 \) defined on \((\Omega, \mathcal{F}_t)\). Moreover \( P_{st}^0|\mathcal{F}_u = P_{su}^0 \), for all \( u \in [s, t] \).
Summary and the following steps.

- Whenever we have a time-consistent family of tame price operators $x_{st}(X)$, $0 \leq s \leq t \leq T$, defined on the whole cone $X \in L_p^+(\mathcal{F}_t)$, we have an EMM.

- This is always true in markets that are complete. However, in general, operators are defined on the sub-cones $L_t^+ \subseteq L_p^+(\mathcal{F}_t)$.

- Then the existence of an EMM is linked to the admissibility of an extension of the price operator from the sub-cones to the corresponding cones.

- Need to give conditions (necessary and sufficient) for the extension of operators.
4. EMM and extension theorems for operators

Let \( m_{st}, M_{st} \in L_q^+(\mathcal{F}_t) \) \((\frac{1}{p} + \frac{1}{q} = 1)\) such that \( 0 < m_{st} \leq M_{st}, P\text{-a.s. and} \)

\[
\begin{cases}
\text{ess sup } E[M_{st}^q|\mathcal{F}_s] < \infty, & 1 < p < \infty, \\
\text{ess sup } M_{st} < \infty, & p = 1.
\end{cases}
\]

**Theorem.** For \( s, t \in [0, T]: s \leq t, \) fixed. The price operator \( x_{st}(X), X \in L_t^+, \)

lying in the *sandwich*

\[
E[Xm_{st}|\mathcal{F}_s] \leq x_{st}(X) \leq E[XM_{st}|\mathcal{F}_s], \quad \forall X \in L_t^+,
\]

admits a tame strictly monotone linear \( \mathcal{F}_s\)-homogeneous extension \( x_{st}(X), X \in L_p^+(\mathcal{F}_t), \) defined on the whole cone \( L_p^+(\mathcal{F}_t) \)

if and only if the *sandwich condition*

\[
E[Y'm_{st}|\mathcal{F}_s] + x_{st}(X') \leq x_{st}(X'') + E[Y''M_{st}|\mathcal{F}_s]
\]

holds for all \( X', X'' \in L_t^+ \) and \( Y', Y'' \in L_p^+(\mathcal{F}_t) \) such that \( X' + Y' \leq X'' + Y''. \)
The *sandwich extension theorem* is an extension theorem for operators lying in a given sandwich. The theorem fits in the Banach lattice framework generalizing the König extension theorem for functionals.

**N.B.** The extension \( x_{st}(X), X \in L^+_p(\mathcal{F}_t) \), if existing, is *sandwich preserving*, i.e.

\[
E[Xm_{st}|\mathcal{F}_s] \leq x_{st}(X) \leq E[XM_{st}|\mathcal{F}_s], \quad \forall X \in L^+_p(\mathcal{F}_t).
\]

**N.B.** The extension, if existing, admits representation

\[
x_{st}(X) = E^0_{st}[X|\mathcal{F}_s] = E[Xf_{st}|\mathcal{F}_s] \quad (f_{st} := \frac{dP^0_{st}}{dP}).
\]

**N.B.** The density \( f_{st} \) lies in the sandwich

\[
0 < m_{st} \leq f_{st} \leq M_{st} \quad P - a.s.
\]
5. A version of the fundamental theorem of asset pricing

Let $m, M \in L_q(\mathcal{F}_T)$ ($\frac{1}{p} + \frac{1}{q} = 1$) such that $0 < m \leq M$, $P$-a.s. and

\[
\begin{aligned}
&\text{ess sup } E[M^q|\mathcal{F}_s] < \infty, \quad 1 < p < \infty, \\
&\text{ess sup } M < \infty, \quad p = 1.
\end{aligned}
\]

For any $s \leq t$ define

\[
m_{st} \defeq \left( E[m|\mathcal{F}_0] \right)^{t-s} \frac{E[m|\mathcal{F}_t]}{E[m|\mathcal{F}_s]}, \quad M_{st} \defeq \left( E[M|\mathcal{F}_0] \right)^{t-s} \frac{E[M|\mathcal{F}_t]}{E[M|\mathcal{F}_s]}.
\]

In particular we have that

\[
m = m_{0T}, \quad M = M_{0T}.
\]

**N.B.** For any $s \leq t$, we have $m_{0T} = m_{0s} m_{st} m_{tT}$. Analogously for $M$. 
Theorem. A tame martingale measure $P^0$ on $(\Omega, \mathcal{F}_T)$ equivalent to $P$ such that the density $f := \frac{dP^0}{dP}$ lies in the sandwich

$$0 < m \leq f \leq M \quad P - a.s.$$ 

exists if and only if for all $s, t \in [0, T]: s \leq t$, the price operators $x_{st}(X)$, $X \in L^+_t$, satisfy the sandwich condition

$$E[Y'm_{st}|\mathcal{F}_s] + x_{st}(X') \leq x_{st}(X'') + E[Y''M_{st}|\mathcal{F}_s]$$

for all $X', X'' \in L^+_t$ and $Y', Y'' \in L^+_p(\mathcal{F}_t)$ such that $X' + Y' \leq X'' + Y''$.

N.B. In this case, we have also

$$0 < m_{st} \leq \frac{E[f|\mathcal{F}_t]}{E[f|\mathcal{F}_s]} \leq M_{st}.$$
Sketch of proof. Necessary condition. Consider the set of EMM:

\[ \mathbb{P} := \left\{ P^0 \mid \frac{dP^0}{dP} = f, \ m \leq \frac{f}{E[f|\mathcal{F}_0]} \leq M; \ \forall s, t \in [0, T], \ s \leq t \right\} \]

\[ \chi_{st}(X) = E^0[X|\mathcal{F}_s], \ \forall X \in L^+_t, \}

and the approximating set:

\[ \mathbb{P}(\mathcal{T}) := \left\{ P^0 \mid \frac{dP^0}{dP} = f, \ m \leq \frac{f}{E[f|\mathcal{F}_0]} \leq M; \ \forall s \in \mathcal{T}, \ t \in [s, T] \right\} \]

\[ \chi_{st}(X) = E^0[X|\mathcal{F}_s], \ \forall X \in L^+_t, \}

where \( \mathcal{T} \) is some partition of \([0, T]\) of the form

\[ \mathcal{T} = \{s_0, s_1, \ldots, s_K\}, \ \text{with} \ 0 = s_0 < s_1 < \cdots < s_K = T. \]
Further, we consider a sequence \( \{T_n\}_{n=1}^{\infty} \) of increasingly refined partitions, such that \( T_n \subset T_{n+1} \) and \( \text{mesh}(T_n) \to 0 \) as \( n \to \infty \). Clearly \( \mathbb{P}(T_{n+1}) \subset \mathbb{P}(T_n) \).

Then the proof proceeds with the following steps:

A. \( \mathbb{P}(T) \) is non-empty for any finite partition \( T \),

B. the infinite intersection \( \bigcap_{n=1}^{\infty} \mathbb{P}(T_n) \) is non-empty, and

C. any \( P^0 \in \bigcap_{n=1}^{\infty} \mathbb{P}(T_n) \) is also in \( \mathbb{P} \).
Example

Let \( L^+ := \{ \alpha X + \beta : \alpha, \beta \geq 0 \} \) be the set of claims, i.e. \( \alpha \) represents the fraction of the claim

\[
X = \int_{z > z_0} (z - z_0)N(T, dz), \quad N(T, dz) \sim Poi(T \nu(dz)),
\]

and \( \beta \) is the amount in a money market account with zero-interest. Note that \( X \) can be interpreted as an insurance policy covering all losses exceeding the deductible \( z_0 \) in the time span \([0, T] \).

Let the price at time 0 of \( X \) be given by the expected value principle, i.e.

\[
x_0T(X) = (1 + \delta)EX = (1 + \delta) T \int_{z > z_0} (z - z_0)\nu(dz).
\]

Let the bounds for the possible density be given by:

\[
m = e^{-\delta T \nu(U)}
\]

\[
M = (1 + \delta)^{N(T, U)} e^{-\delta T \nu(U)}, \quad U := (z_0, \infty)
\]
Then both

$$P_1^0\{N(T, dz) = n\} = (1 + \delta)^n e^{-\delta T \nu(dz)} \cdot P\{N(T, dz) = n\}$$

and

$$P_2^0\{N(T, dz) = n\} = \begin{cases} 1 \cdot P\{N(T, dz) = n\}, & z \leq z^*, \\ (1 + \gamma)^n e^{-\gamma T \nu(dz)} \cdot P\{N(T, dz) = n\}, & z > z^*, \end{cases}$$

for $z^* > z_0$ and $\gamma := \frac{\delta \mathbb{E}X}{T \int_{z^*}^{\infty} (z - z_0) \nu(dz)}$, are EMM.

However, the sandwich condition shows that while for $P_1^0$ we have

$$m \leq \frac{dP_1^0}{dP} \leq M \quad P \text{ – a.s.,}$$

for $P_2^0$ the relation is not true.
This presentation was based on:
G. Di Nunno, Inga B. Eide. Events of small but positive probability and a fundamental theorem of asset pricing. E-print, University of Oslo 2008.

Relevant related references:


