DYNAMICAL SYSTEMS METHOD (DSM) FOR SOLVING OPERATOR EQUATIONS

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Abstract

A general method, Dynamical Systems Method (DSM), for solving nonlinear and linear operator equations $F(u) = f$ in Hilbert space is presented. This method consists of the construction of a dynamical system, that is, a Cauchy problem,

$$\dot{u} = \Phi(t, u), \quad u(0) = u_0,$$

which has the following three properties:

$$\exists! u \forall t \geq 0; \quad \exists u(\infty); \quad F(u(\infty)) = f.$$

Various choices of nonlinear map $\Phi(t, u)$ are proposed and the DSM is justified for wide classes of operator equations, including

a) arbitrary solvable linear operator equations of the form $Au = f$ with densely defined closed linear operator A,

b) well-posed nonlinear equations,

c) ill-posed nonlinear equations with monotone operators,

d) operators F such that $A := F'(u)$ satisfies some spectral assumption.

Convergence theorem is obtained for a DSM version of Newton's method for monotone operators for any initial approximation.

A general approach to constructing convergent iterative schemes for solving well-posed nonlinear operator equations is described and convergence theorems are obtained for such schemes. Stopping rules for stable solution of ill-posed problems with noisy data are given.

References

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