UNDERSTANDING, MODELING AND MANAGING LONGEVITY RISK.

Stéphane Loisel, ISFA- U. Lyon 1

Joint work with the members of the research chair on longevity risk with financial support of Fédération Bancaire Française:
Pauline Barrieu (LSE), Harry Bensusan (CMAPX), Nicole El Karoui (CMAPX), Caroline Hillairet (CMAPX), Yahia Salhi (ISFA and SCOR)

Jena - March, 2011
http://hal.archives-ouvertes.fr/hal-00417800


**FIGURE:** Migrations and life expectation

<table>
<thead>
<tr>
<th>age x = 60</th>
<th>male</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>population</td>
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<tr>
<td>x_60</td>
<td>19.35</td>
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**FIGURE:** Migrations and life expectation

<table>
<thead>
<tr>
<th>male</th>
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<th>male pens</th>
<th>difference</th>
<th>percentage difference</th>
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<td>25.83</td>
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Introduction
Characteristics of longevity risk
Modeling longevity risk
Longevity risk and new regulations
Transferring longevity risk
Modeling issues for pricing
Basis Risk
Detection of drift changes

**Figure**: Migrations and life expectation

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<th>Difference</th>
<th>Percentage Difference</th>
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<tbody>
<tr>
<td>e_x</td>
<td>20.39</td>
<td>26.56</td>
<td>6.16</td>
<td>30.2</td>
</tr>
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</table>
French national population

Figure 4  Log-mortality structure of French male, 1962-2000
$\log \mu_{x,t} = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim \mathcal{N}(0, \sigma),$
Longevity improvements at different ages

Figure 3.a: $b_x$ schedule for the entire time series 1947-1999 and two selected subsamples 1947-1970 and 1976-1999, female

Examining Structural Shifts in Mortality Using the Lee-Carter Method
Lawrence R. Carter, Alexia Prskawetz
PURE LONGEVITY RISK

- change of the average trend
- short-term oscillations around the average trend (risk of over-reactions)
- Heterogeneity and basis risk: the evolution of the policyholders mortality is usually different from that of the national population (selection effects).

Financial Risk

- Long term interest rate risk
- Counterparty risk
Part of systemic risk by maturity for 8000 policyholders

Part of variance due to systemic risk in the variance of the sum of discounted cashflows (real portfolio)

Percentage

Number of years where annuities are paid

- Green box
- Blue iid
- Red lie
## Introduction

### Characteristics of Longevity Risk
- Life tables
- Period and cohort tables and effects
- Heterogeneity and basis risk

## Modeling Longevity Risk

## Detection of Drift Changes

## Longevity and Mortality Risks: A Natural Hedge?

## Transferring Longevity Risk

## Modeling Issues for Pricing
- Pricing methodologies
- Long-term interest rates
Mortality analysis of a population or an insured portfolio depends on the available data and their reliability. These data are based on statistics coming from various national institutes (INSEE in France, Bureau of Census in the US, CMI in the UK, etc.), available through the Human Mortality Database. The life table is a decreasing sequence of the estimated number of people alive at date $t$ and at given age $x$ from an initial group of individuals.

- periodic life tables
- cohort life tables

Classical life tables are well-suited to quantify short-term mortality risk (death insurance), for time horizons from 1 to 5 years provided that no exceptional event occurs (such as pandemic or heat wave).

BUT these tables are not relevant for long term longevity-based contracts like annuities or pensions, as mortality rates are changing over time and one must take this evolution into account.
This is an age-year-cohort diagram representing the evolution of mortality over time. The real cohort mortality is followed on a diagonal manner and the fictitious (also called period) cohort could be read vertically. For example, the black circle corresponds to death of an individual, born in 1943, in 2008. The circle is situated on the upper triangle of the death year 2008 box (the grey box), meaning that the individual died in late 2008, say at age 65.60.
Because of the fictitious nature of period life tables, one must be very cautious with period-based longevity indices to avoid over-reactions and as a consequence basis risk.

As example, let us compare retrospective life expectancy at birth for English and Welsh males obtained from period tables and cohort tables from the period 1840 to 1925.

**Figure**: Periodic life expectancy (left) and cohort life expectancy (right) at birth in the UK.
HETEROGENEITY AND BASIS RISK

Difference between the national mortality data and the one from an insured portfolio.

- Insurance companies have much more detailed information
  - They know the exact ages at death and not only the year of death (time continuous data)
  - Cause of death are specified
  - characteristics of the policyholders: socio economic level, living conditions ...

- BUT
  - limited size of their portfolios (in comparison to national populations: 700,000 individuals from 19 different insurance companies)
  - small range of the observation period
Longevity patterns and longevity improvements are very different from one company portfolio to the other, and even for different countries. Factors affecting the mortality

- socio-economic level (occupation, income, education...)
- gender
- living environment (pollution, nutritional standards, hygienic...)

Furthermore, insurers are tending to select individuals (given their health and medical history for example).

This heterogeneity is very important for longevity risk transfer, as basis risk may be too important for insurers to accept to use financial instruments based on national indices to hedge their longevity risk, as this hedge would be too imperfect.
Co-integration or not?
Stability in time?

Figure 1  Mortality on logarithmic scale for both insured population (left) and national population (right) at different ages: 60 (solid), 70 (dashed), 80 (dotted) and 90 (dotdash).
INTRODUCTION

CHARACTERISTICS OF LONGEVITY RISK
- Life tables
- Period and cohort tables and effects
- Heterogeneity and basis risk

MODELING LONGEVITY RISK

DETECTION OF DRIFT CHANGES

LONGEVITY AND MORTALITY RISKS: A NATURAL HEDGE?

TRANSFERRING LONGEVITY RISK

MODELING ISSUES FOR PRICING
- Pricing methodologies
- Long-term interest rates
Lee-Carter Model (1992)

This model describes the force of mortality, $\mu_{x,t}$ at age $x$ and time $t$ by three series of parameters namely $\alpha_x$, $\beta_x$ and $\kappa_t$ as follows:

$$\log \mu_{x,t} = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim \mathcal{N}(0, \sigma),$$

- $\alpha_x$ gives the average level of mortality at each age over time
- $\kappa_t$ is the general speed of mortality improvement over time
- $\beta_x$ is an age-specific component that characterizes the sensitivity to $\kappa_t$ at different ages
- $\varepsilon_{x,t}$ captures the remaining variations
- + constraints on the parameters to enforce the uniqueness: $\sum \beta_x = 1$ and $\sum \kappa_t = 0.$
The general model gives the dynamic of the annual mortality rate $q_t(x)$ at age $x$ during the year $t$:

$$\text{logit}q_t(x) = \kappa_1^t \beta_1^t \gamma_1^{t-x} + \cdots + \kappa_n^t \beta_n^t \gamma_n^{t-x}.$$ 

Three types of parameters:

- $\beta^i$ specific to age
- $\kappa^i$ specific to calendar year
- $\gamma^i$ cohort effect parameters
A particular example of the CDB model, featuring both the cohort effect and the age-period effect:

$$\text{logit}\mu_{x,t} = \kappa_1^t + \kappa_2^t (x - \bar{x}) + \kappa_3^t \left( (x - \bar{x})^2 - \sigma_x^2 \right) + \gamma_{t-x},$$

- $\bar{x} = \frac{\sum_{x=x_0}^{x_n} x}{x_n-x_0+1}$ is the mean age of the historical mortality rates,
- $\sigma^2$ is the standard deviation of ages, equal to $\frac{\sum_{x=x_0}^{x_n} (x-\bar{x})^2}{x_n-x_0+1}$,
- $\kappa_1^t, \kappa_2^t$ and $\kappa_3^t$ correspond respectively to the general mortality improvement over time, the specific improvement for every age and the age-period related coefficient,
- $\gamma_{t-x}$ represents the cohort-effect component.
MICRO-MACRO MODELING FOR LONGEVITY RISK

(Harry Bensusan’s PhD thesis):
Use population dynamics methods to take characteristics of policyholders and their evolution into account.
MATHEMATICAL FRAMEWORK

Let \( \{\kappa(t), t \in \mathbb{R}\} \) be the Lee-Carter time component whose dynamic satisfies:

\[
d\kappa(t) = \xi s^{-\frac{1}{2}} f\left(\frac{t - t_0}{s}\right) dt + dW(t),
\]

where

- \( t_0 \in C \subset \mathbb{R} \), \( \xi \) is non-negative parameter representing the amplitude and \( s \) is a positive parameter representing the scale of the trend.
- \( f \) is a square integrable function.
- \( W \) is a Brownian motion.

OBJECTIVE

*Testing the null hypothesis of no trend component: \( H_0 : \xi = 0 \), against \( H_1 : \xi \neq 0 \). We use the method of Siegmund and Worsley (1995).*
1 INTRODUCTION

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3 MODELING LONGEVITY RISK

4 DETECTION OF DRIFT CHANGES

5 LONGEVITY AND MORTALITY RISKS: A NATURAL HEDGE?

6 TRANSFERRING LONGEVITY RISK

7 MODELING ISSUES FOR PRICING
   - Pricing methodologies
   - Long-term interest rates
Can we "buy" mortality risk in order to hedge longevity risk?

Even if a certain mutualization between mortality and longevity risks obviously exists, it is very difficult to obtain a significant risk reduction between the two, because of their different natures:

- Mortality risk is a short-term risk (1 to 5-year maturity) with a catastrophic component (pandemic, heat wave, ...)
- Longevity risk is a long-term risk with maturities ranging from 20 to 80 years and is mainly about changes in the trend.

The impact of a pandemic or a catastrophe on mortality is really different from the impact on longevity: an abnormally high death rate at a given date has a reduced influence on the longevity trend.

Remember to talk about the 2003 heat wave and recent mortality bonds.
Solvency Capital Requirements (SCR)

In Solvency II, the SCR is computed separately for each risk factor when stressing those risk factors (for example for longevity risk, a 10% decrease on mortality rates each year).

\[ SCR_i = \text{VaR}_{\alpha}(M_i) - \mathbb{E}(M_i), \]

where \( \alpha = 99.5\% \) and \( M_i \) are the liabilities.

The global SCR is computed by aggregating each single \( (SCR^j)_j \):

\[ \text{SCR}_{\text{global}} = \sqrt{\sum_{i>0} \sum_{j>0} \theta_{i,j} \text{SCR}_i \text{SCR}_j}. \]

"Correlation" parameters \( (\theta_{i,j})_{i>0,j>0} \) are pre-defined by the regulator. The only negative one (-25%) is the one between longevity and mortality risks.

New regulations \( \Rightarrow \) increase of solvency requirements \( \Rightarrow \) need for capital
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Insurance-Linked Securitization:

- complement the reinsurance industry in catastrophic risk management (Goshay and Sandor (1973))
- use of capital markets to transfer insurance risk

ILS market has been growing very fast over the last 15 years

- The non-life part of the ILS market is the most visible with the famous and highly successful cat-bonds
- The life part of the ILS market is the bigger in terms of volume of the transactions with an estimated outstanding of 35 to 40 billion USD
No ILS related to longevity risk has been completed yet BUT

- the estimation of the underlying public and private exposure on longevity risk is over 20 trillions USD
- The Life and Longevity markets association (LLMA) launched on February 2010 to promote trading of longevity risk (established by AXA, Deutsche Bank, JP Morgan, Legal & General, Pension Corporation, Prudential, RBS and Swiss Re).

Obstacles to develop capital markets’ solutions

- one-way exposure of investors: there is almost no natural buyers of longevity risk → creates a problem to generate demand
- basis risk: heterogeneity between full population mortality indices and those of individual pension funds and insurers, regional and socio-economic basis risk...
A good longevity index should be based on national data (for transparency) but be flexible enough as to reduce the basis risk for the original longevity risk bearer. Today, the existing indices are:

- Credit Suisse Longevity Index (December 2005) based upon national statistics for the US population, with some gender and age specific sub-indices.


- Goldman Sachs Mortality Index (December 2007) based on a sample of US insured population over 65 and targets the life settlement market.

- Xpect Data by Deutsche Borse (March 2008) initially delivered monthly data on life expectancy for Germany, but now covers the Netherlands.
Q-FORWARDS

JP Morgan has developed some standardized longevity instruments called "q-forwards".

- contracts are based upon an index (the mortality rate or the survival rate, as quoted in LifeMetrics).
- The mechanisms of the q-forwards are quite simple: a pension fund hedging its longevity risk will expect to be paid by the counterpart of the forward if the mortality falls by more than expected.

  a pension fund = q-forward seller
  an investor = q-forward buyer
LONGEVITY SWAP TRANSACTIONS

Very recently, some longevity swap transactions have been completed: private transactions and their pricing remains confidential. Over the last year 2008, two particular longevity swaps have been arranged by JP Morgan:

- A customized swap transaction (July 2008), notional amount of GBP 500 millions for 40 years. The UK life insurer pay fixed payments and receive floating payments which replicates the actual benefit payments made on a closed portfolio of retirement policies (no basis risk).

  At the same time, JP Morgan entered into smaller swaps with several investors who take the longevity risk at the end. The counterpart risk for this swap is important because of the long term maturity and the number of agents involved.

- A standardized transaction (January 2008), notional amount of GBP 100 millions for 10 years. standardized longevity swap with the pension insurer Lucida, using LifeMetrics index for England and Wales as underlying index (basis risk for Lucida).
The challenge lies in developing transparency and liquidity without neglecting the hedging purposes of the instruments.

- essential challenge: designing suitable, efficient and attractive structures for both risk bearers and risk takers (as underlined by the failure of the EIB-BNP Paribas longevity bond in 2005).
- Emphasizing the importance of assessing counterpart risk, to secure transactions (even more critical when considering longevity risk, due to the long-term maturity)
- question of the pricing of risk transfer solutions
Insurance companies are exposed to interest rate risk

- Annuities at rate $k$
- Investments in interest rate products (Bonds,...)

Looking for a product that transfers interest rate risk while keeping longevity risk:

- Insurance can manage longevity risk
- Banks can manage interest rate risk

⇒ Product with decorrelation of both risks
Difficulties for launching a pure longevity product:

- No longevity market (No shared reference)
- Information asymmetry
- Modelling and pricing (evaluation in historical probability)

⇒ Pure interest rate products
FIGURE: Age distribution of policyholders
**PORTFOLIO EVOLUTION**

*Figure*: Survival extreme scenarii of policyholders

Using our longevity model
Interest rate hedging of life-insurance products:

- Static hedge of interest rate risk ⇒ Swaps
- Dynamic hedge of forward risk ⇒ Swaptions
- Longevity ⇒ Swaptions with variable nominal
- Stochastic evolution of longevity ⇒ Choice of the nominal profile
Swaption on a swap with variable nominal $N_t$ with strike $k$:

$$
\frac{P_{swaption}}{\delta B(0,T)} = \mathbb{E}^{Q_T} \left[ (k - SV_T(T_0,T_N,\delta,N_t))^+ \sum_{i=1}^{N} B(T,T_i)N_{Ti} \right]
$$

Choice of $\alpha_T \in [0,1]$ by the insurer at date $T$ (with available information)

$\Rightarrow$ Hedge on the nominal series $N_t^{\alpha_T} = \alpha_T N_t^- + (1 - \alpha_T) N_t^+ $

Forward swap rate with variable nominal $SV_T(T_0,T_N,\delta,\alpha_T,N_t^-,N_t^+)$ for the series $N_t$ determined by $\alpha_T$

Evaluation of "Life Nominal Choosing Swaption" (LNCS) at strike $k$:

$$
\frac{P_{LNCS}}{\delta B(0,T)} = \mathbb{E}^{Q_T} \left[ \max_{0 \leq \alpha_T \leq 1} \left\{ (k - SV_T(T_0,T_N,\delta,\alpha_T,N^-_t,N^+_t))^+ \sum_{i=1}^{N} B(T,T_i)(\alpha_T N^-_{T_i} + (1 - \alpha_T) N^+_{T_i}) \right\} \right]
$$

$\sim \mathbb{E}^{Q_T} \left[ \max_{0 \leq l \leq n} \left\{ (k - SV_T(T_0,T_N,\delta,\frac{l}{n},N^-_t,N^+_t))^+ \sum_{i=1}^{N} B(T,T_i)(\frac{l}{n} N^-_{T_i} + (1 - \frac{l}{n}) N^+_{T_i}) \right\} \right] $
**CHOICE OF $\alpha_T$ (EXAMPLE)**

**Figure:** Choice of the parameter $\alpha_T$ in 2019

Using our longevity model
CHOICE OF $\alpha_T$ (EXAMPLE)

**Figure: **Choice of the parameter $\alpha_T$ in 2029

Using our longevity model
PROJECTION SCENARIOS

- Double stochasticity (mortality process and evolution process)
  ⇒ various scenarii

- Approximation to large population ⇒ relevant behaviour

- Average scenario is realistic and close to INSEE projections

⇒ Identification of the scenarii panel (extreme or not) generated by the model
Product on maximum: impact of rates correlation

- 2-factor HJM calibrated on European rate curve and on specific swaptions (maturity and tenor adapted)
- Assume constant correlation between successive forward rates:

\[ \rho = \text{correl}(f(., T), f(., T + dt)) \]

- Focus on price impact of \( k \) and \( \rho \)
**INITIAL RATE CURVE**

**Figure**: European rate curve as of 8 April 2009
NOTION OF COST ON ANNUITIES

- **Annuities:**

\[ P_{\text{annuities}} = k \times LVL(\alpha), \]

where \( LVL(\alpha) = \sum_{i=1}^{N} (\alpha N_{T_i}^- + (1 - \alpha) N_{T_i}^+) B(T, T_i) \)

- **Life Nominal Choosing Swaption:**

\[ P_{\text{LNCS}} = c \times LVL(\alpha) \]

\[ \Rightarrow c = \frac{P_{\text{LNCS}}}{LVL(\alpha)} \text{ called cost on annuities} \]

- **Interpretation:**
  - Without product: annuities at rate \( k \) without hedging interest rate risk
  - With product: annuities at rate \( k + c \) with hedging interest rate risk
**Correlation Impact**

<table>
<thead>
<tr>
<th>Successive rate correlation</th>
<th>Swap correlation 10Y/12Y</th>
<th>Price $\alpha = 0$</th>
<th>Price $\alpha = 0.5$</th>
<th>Price $\alpha = 1$</th>
<th>Price LNCS</th>
<th>Cost on Annuities</th>
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<tr>
<td>$\rho = 0.998$</td>
<td>99.6%</td>
<td>3015 bp</td>
<td>2656 bp</td>
<td>2244 bp</td>
<td>3020 bp</td>
<td>0.87%</td>
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<td>$\rho = 0.997$</td>
<td>99.3%</td>
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<td>2580 bp</td>
<td>2215 bp</td>
<td>2960 bp</td>
<td>0.853%</td>
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<td>2732 bp</td>
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<td>2751 bp</td>
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<td>98.3%</td>
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<td>2342 bp</td>
<td>2061 bp</td>
<td>2667 bp</td>
<td>0.769%</td>
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<td>$\rho = 0.992$</td>
<td>98%</td>
<td>2540 bp</td>
<td>2264 bp</td>
<td>2009 bp</td>
<td>2574 bp</td>
<td>0.742%</td>
</tr>
</tbody>
</table>

**Table:** Evolution of the price at $k = 4.3\%$ depending on successive rate correlation

When $\rho$ decreases

- Price of swaptions with variable nominal decreases

- Price of the switch option increases from $3020 - 3015 = 5bp$ to $2574 - 2540 = 34bp$

$\Rightarrow$ exotic nature increases but remains low

H. Bensusan (Société Générale)  The Life Nominal Choosing Swaptions
CONCLUSIONS

- Estimation of basis risk: portfolio heterogeneity
- Pure interest rate product suited to expectations of banks/insurance companies
- Could use more sophisticated model for interest rate