Variable annuities: confronting points of view of insurers and bankers, with an emphasis on surrender risk

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Table of Contents

1 Types of guarantees offered in Variable Annuities

2 Risk analysis and Solvency II pitfalls

3 Confronting points of view of insurance and finance
Types of guarantees offered in Variable Annuities

- Types of contracts
- Guarantees and surrender risk

Risk analysis and Solvency II pitfalls

Confronting points of view of insurance and finance
The ancestor of VA’s in France

- **Garanties plancher** in unit-linked life insurance contracts
- Corresponds to a GMDB product.
- Sum of payments is guaranteed for the beneficiary if policyholder dies.
- Used to be a compulsory guarantee in France.
- Usually priced by insurers thanks to *la formule des puts pondérés*. 
More sophisticated guarantees

- Return of Premium: higher of total premium or account value, adjusted for withdrawals
- Roll-up: premiums paid accumulated at guaranteed rate, adjusted for withdrawals
- Ratchet: highest account value at contract anniversary dates, adjusted for withdrawals
- Greater of Ratchet or Roll-up: greater of annual ratchet or roll-up amount paid to successors if policyholder dies.
Roll-up and Ratchet Guarantees

Option: Roll-up 5%

Option: Ratchet annuel
Guarantees can be combined
Types of guarantees offered in Variable Annuities

- Types of contracts
- Guarantees and surrender risk

Risk analysis and Solvency II pitfalls

Confronting points of view of insurance and finance
GMIB vs GMWB types

- GMIB guarantees a minimum annual income as long as the policyholder is alive. After the end of deferral period, the Account Value is converted into a reserve on the general account of the insurance company. This reserve is used to pay incomes. After deferral period, policyholder cannot exit the contract and does not own Unit-Linked.

- GMWB guarantees a minimum annual withdrawal for a given period or for life. After the end of deferral period, the policyholder still owns her Account Value. After deferral period, policyholder can exit the contract and recover his Account Value minus received withdrawals.
GMWB

UC

Montant de la Garantie

\[ t=0 \quad \text{retrait} \quad \text{retrait} \quad \text{échéance} \]
Types of guarantees offered in Variable Annuities

Guarantees and surrender risk

GMWB

[Diagram showing the evolution of the guarantee and surrender risk over time]

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More and more sophisticated products in the US

<table>
<thead>
<tr>
<th>1980s</th>
<th>90-95</th>
<th>95-00</th>
<th>Post 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• GMDB</td>
<td>• Introduction of GMIB’s</td>
<td>Garantees</td>
</tr>
<tr>
<td></td>
<td>• Fees can be paid regularly</td>
<td>• Reduction of margins</td>
<td>• GMAB</td>
</tr>
<tr>
<td></td>
<td>• 0 Penalty</td>
<td>• Sophistication of products</td>
<td>• GMWB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Introduction of ratchet guarantees for GMDB’s</td>
<td>• GMLB</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sometimes no surrender penalty</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Embedded disability insurance in some VA’s</td>
</tr>
</tbody>
</table>

Start in Europe

1980s - 1990s
90-95
95-00
Post 2000
Potential market (predicted in 2008!)

Estimated market size in Europe
(AEGON – Estimated assets; €Mds)

UK
70 - 170

Netherlands
10 - 30

Germany
40 - 90

Spain
15 - 35

France
80 - 185

Italy
45 - 110

CEE
10 - 30

1. Source: AEGON (Analyst conference 2/3 June 2008)
VAs in Europe

Launched i2Live in May 2007

First mover in UK ‘Gold’ and ‘Platinum’
Was market leader
Now withdrawn

European roll-out of ‘Accelerator’ plans in Germany (‘Twinstar’), Belgium, France, Spain, Italy, UK Offshore (Isle of Man), Portugal, Switzerland

DECP - 3
Emergence of the Irish platform

Twinstar – Germany

Accumulator – Spain

Generation F – Spain

Twinstar – Belgium

Global TopReturn – Germany

Münchener Rück Munich Re Group

Active Rispamio – Italy

Capital Resources – France

Accumulator Retraite – France

Allianz

AG2R LA MONDIALE – France

SwissLife

Q4 – France

SwissLife

Q4 – Germany

The Hartford

SafetyNet

Estate planning bond

AEGON

5 for Life

Riley bond

The Hartford

Platinum

Royal London

Secure Retirement

MetLife

In Ireland

i2Live

Lincoln Financial Group

Investment Control

AEGON

Income for Life

AIG

Q1 – UK

Q4 – UK

DECP - 4
The effect of the crisis

Some players withdraw some products (like AXA in Germany)

Some others stop their VA business (Hartford in Europe)

Regulators (Ireland, France, Germany,...)

2010: the VA market restarts as if nothing happened?
1. Types of guarantees offered in Variable Annuities

2. Risk analysis and Solvency II pitfalls
   - Some key risks associated to VA’s
   - Risk aggregation in Solvency II internal formula

3. Confronting points of view of insurance and finance
Some key risks associated to VA’s

VA’s key risks

- Volatility, interest rates, Gamma, exchange rates
- Basis risk
- Arbitrage (investment choices made by policyholders)
- Policyholders’ behavior risk (surrender, arbitrage)
- Liquidity
- Counterparty, Key-Person, operational
- Transaction costs, cost of capital
- ...

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Solvency Capital Requirement (SCR)

Standard formula: Lognormal loss distribution assumption
1-Year Time Horizon

Figure (Source: CP 20, p. 9)
2.2 Pillar 1 is made up of a number of difficult combinations, should provide a structured indication of whether the insurer has adequate financial resources.
Risk aggregation

$$SCR_{glob} = \sqrt{\sum_i \sum_j \rho_{ij} SCR_i SCR_j}.$$ 

The $\rho_{ij}$ are correlation parameters given by QIS4 matrices.

Advantage of the above formula: it takes diversification effect into account: the global SCR is always smaller than the sum of the SCR per LoB.

Some obvious drawbacks:
- multi-level risk aggregation problems
- Gap with internal models

Incentive to develop internal risk models.
Multi-level risk aggregation: exercise

Suppose we have the following risks and required capital for each risk on a stand alone basis:

\[ SCR_{glob} = \sqrt{\sum_i \sum_j \rho_{ij} SCR_i SCR_j}. \]

<table>
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<td>1000</td>
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<tr>
<td>B</td>
<td>200</td>
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<tr>
<td>C</td>
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And the following correlation factors between those risks:

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1) Compute the risk capitals for A+B, for C+D and for A+B+C+D.
2) Derive the implicit correlation coefficient between (A+B) and (C+D).
Multi-level risk aggregation: solution

Suppose we have the following risks and required capital for each risk on a stand alone basis:

\[ SCR_{glob} = \sqrt{\sum_i \sum_j \rho_{ij} SCR_i \cdot SCR_j}. \]

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And the explicit correlation factor between \((A+B)\) and \((C+D)\)

\[ 0.865 = \left( \frac{3192^2 - 1114^2 - 2179^2}{2 \times 1114 \times 2179} \right) \]

1) Compute the risk capitals for A+B, for C+D and for A+B+C+D.
2) Derive the implicit correlation coefficient between \((A+B)\) and \((C+D)\).
Multi-level risk aggregation: exercise

Suppose the correlation factors stay the same but during a year the capitals change from the ones in table 1 into (this could arise from changes in business volume or of product mix):

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1) Recompute the risk capitals for A+B, for C+D and for A+B+C+D.
2) Derive the new implicit correlation coefficient between (A+B) and (C+D).
Multi-level risk aggregation: solution

Suppose the correlation factors stay the same but during a year the capitals change from the ones in table 1 into (this could arise from changes in business volume or of product mix):

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<td>1277</td>
</tr>
<tr>
<td>C+D</td>
<td>2145</td>
</tr>
<tr>
<td>A+B+C+D</td>
<td>3336</td>
</tr>
</tbody>
</table>

From these capitals we can derive that while the dependencies between the risks did not change at all the correlation factor between (A+B) and (C+D) changed from: **0.865** into **0.895** to get the total capital for A+B+C+D: 3336.
Multi-level risk aggregation: conclusion*

Only correlation factors set at the lowest level are unequivocal and stable over time. Thus capitals at higher levels should always be derived starting at a lower level. Deriving a higher level aggregation from a lower level aggregation above the sub risk level risks will result in misleading outcomes at the entity or group level.

So, the Bottom up approach is

Surrender risk

- What’s new?
  Best estimates assumptions instead of prudential assumptions defined today by the Code des Assurances (in France);

- What does it change?
  Today, estimated surrender rate is 100% in technical reserves: the insurer is forced to reserve at least the mathematical reserve of the contract. (the policyholder can surrender her contract whenever she wants)

  In Solvency II, the reserves may be adapted to the observed surrender rate. The insurer is allowed to reserve only what corresponds to the observed / modeled surrender probability.

- Thus, surrender assumptions

  Nowadays : impacts ALM predictions and EEV;

  In the future : will entirely take part in Accounting of the company!
SCR requirements

- SCR stress tests: QIS 5 report suggests closed formulas to model surrenders (depending on guarantees, credited rates, index performance).
- but this has to be calibrated with the portfolio;
- Lapse risk is part of the underwriting risk module (50% correlated with the SCR linked to management fees drift...);
- In the last technical pre-specifications, the insurer should compute the impact on the SCR of
  - a constant drop by 50% of the lapse rate (limited to an absolute drop of 20%),
  - a constant rise by 50% of the lapse rate (limited to 100%),
  - massive surrenders of 30% of the population-at-risk (absolute increase).
- The biggest SCR is then inserted into the correlation matrix.
4 main approaches to model surrender rates

- **Financial approach**: pricing the surrender option, no behavioral model. Policyholders are assumed to behave like optimal investors. Huge amount of papers.

- **Statistical approach**: collective. Empirical data enables one to calibrate the following surrender function;

- **Economical approach**: individual. Microeconomy, expected utility theory.

- **Probabilistic / econometric approach**: individual. Segmentation models, to define risk classes on a given population in the classical model (GLM, CART and others, see Milhaud, L. and Maume-Deschamps (2010)).
1. Types of guarantees offered in Variable Annuities

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3. Confronting points of view of insurance and finance
   - Pricing methodologies
   - Optimal behavior vs lapse rates
   - Surrender risk
Actuarial vs financial

In general,

- Actuaries tend to price insurance contracts as (Best Estimate of Liabilities + Risk Margin).
  The Risk Margin is theoretically supposed to be computed thanks to a cost of capital approach.
  The Risk Margin should be different from one company to the other (depends of the overall risk profile and objectives in principle).

- In finance, one often uses risk-neutral pricing, indifference pricing, ...

- Concepts are similar: a risk premium is present in both cases.

- But insurers too often forget that risk neutral pricing required that the risk is hedged!
Actuarial and financial pricing may be mixed

- Basic example: Guaranteed Minimum Death Benefit
- Insurers typically use the so-called Weighted Put Formula.
- Correlation between pandemic risk and market risk is ignored.
- Pricing method is subject to caution (risks cannot be duplicated).
Optimal behavior vs lapse rates

- Optimal behavior (financial approach): Quanto-Asian options (see Milevsky’s paper: Why your grandmother purchases Quanto-Asian options)
- Complex problems, in particular some of them are related to exercise frontier computations of American options
- In real-world, policyholders do not behave like that. But what if consultants start to train policyholders in the future?
- The most important question: are investment decisions made by the policyholders, or are they strongly suggested by contract vendors?
References on general VA pricing and hedging framework

- Mortality risk: Schrager[05], Plat[09]; Moller[98,01], Milevsky and Posner[01], Biffis et al. [01, 05, 09].
- Stochastic volatility: Ballota and Haberman[03], Chu and Kwok[07], Haastrecht, Plat and Pelsser[09].
- Stochastic interest rate: Bacinello and Ortu[94], Nielsen and Sandmann[95], Miltersen and Persson[99], Coleman, Li and Patron[04].
References on risk management of VA’s

- Milevsky and Salisbury [06]: financial valuation of GMWB’s.
- Taksar and Hunderup [07]: optimal proportional reinsurance strategies.
- Dai, Kwok and Zong [08]: optimal withdrawal strategies in GMWB’s.
- Forsyth et al. [02,05,07]: Numerical methods.
- Kling, Ruez and Russ [10]: the impact of stochastic volatility on pricing, hedging and hedge efficiency of variable annuity guarantees.
- For detailed mathematical framework, please see T.-L. Nguyen’s slides (Cours Bachelier 2011).
Who makes the decision?

- Kanheman-Tversky paradox
- Different views: arbitrage and surrender vs surrender
- An example of fallacious conclusion
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How can independent risks become suddenly strongly correlated?

Endogenous uncertainty:

- Uncertainty is generated/modified by response of individual entities to events
- Feedback loop: outcomes $\rightarrow$ forecasts $\rightarrow$ decisions $\rightarrow$ outcomes $\rightarrow$ revised forecasts $\rightarrow$ revised decisions $\rightarrow$ ... (Millennium Bridge)
- Statistical relationships are endogenous to the model, and may undergo structural shifts (Goodhart’s Law: *Any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes*)
- Relevant when individual entities are similar in terms of forecasts and likely reactions to events
- Relevant when outcomes are sensitive to concerted actions
How can independent risks become suddenly strongly correlated?

**Endogenous Risk**
- Risk from shocks generated and amplified *within* the system

- in contrast to...

**Exogenous Risk**
- Risk from shocks from *outside* the system

**Millennium Bridge**
- New design
- Tested with extensive simulations
- All angles covered
- No endogenous shocks

- Pedestrians had some problems
- Bridge closed
How can independent risks become suddenly strongly correlated?

Diagnosis

- Trouble at 1 hertz (one complete cycle per second)
- Walking pace: two steps per second (2 hertz)
- But sideways motion every two steps (1 hertz)
How can independent risks become suddenly strongly correlated?

Bridge moves → Adjust stance

↑ Further adjust stance ↓ Push bridge
Analogy with LTCM

Feedback Revisited

Margin calls $\rightarrow$

↑

Distress $\rightarrow$ Adverse price move

Unwind leveraged trades $\downarrow$

This analogy is drawn from Danielsson and Shin (2003). Potential massive surrenders (Loisel and Milhaud (2010), Arnal et al. (2010)) in life insurance, link with Paul Embrechts’s ICA 2010 talk.
Surrender rate VS delta_r

Region 1

Region 2

Distribution change

Lyon - 22/07/2010