New Challenges between Finance and Insurance,

The Longevity Risk

A Microscopic Modeling Approach

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   - Longevity Market, and Risk Management
   - Pension Funds

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Introduction
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What is Longevity Risk?

- **For Insurance or Financial Market** A liability has a longevity risk exposure whenever cash flows are guaranteed for the lifetime of a recipient.

- **For Retirees** The risk that the amount of money an individual saves for retirement might not be enough to sustain them, due to increased life expectancy.

- **Systemic Risk?** Longevity risk has a very long tail, in line with the human expectancy. Largely systemic risk which runs off slowly over time.
Improvements in longevity are bringing new issues and challenges at various levels: social, political, economic and regulatory.

- **Hedging longevity risk** is now an important element of risk management for many organisations

- **The capital markets** are developing as an alternative channel for longevity hedging
  - Complementary to the insurance markets
  - Provide additional capacity and potential for liquidity
Longevity Risk: Subject for Pension Funds

- Many companies (especially in US) have closed the **defined benefit retirement** plans that they offered to their employees.
- In several countries, defined benefit pension plans have been continuously replaced with **defined contribution plans**.
- Some governments are about to increase the retirement age by 2 or 5 years.
UK Market Exposure

Global Market: $25 trillion

▶ UK Market: UK government liabilities exceed 2000bn
  ● State pensions: 1170bn
  ● Unfunded: 770bn
  ● Local authority: 160bn

▶ Defined Benefit pension plans Total pension liabilities = 1000bn, of which pensions in payment = 500bn

▶ Annuity providers: 125bn

▶ Defined Contribution plans: Total assets 450bn
  Of which over age 55 = 150bn
The insurance industry is also facing some specific challenges related to longevity risk.

- Need of more and more capital to face this long-term risk
- Important to find a suitable and efficient way to cross-hedge or to transfer part of the longevity risk to reinsurers or to financial markets.
- Accurate longevity projections are delicate (prospective life tables)
- Modeling the embedded risk (such long term interest rate risk) remains challenging
1 Introduction

2 Characteristics of longevity risk
   - Life tables
   - Period and cohort tables and effects
   - Heterogeneity and basis risk

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Mortality rate by age 1900-2004

Figure 1. 1900-2004 Mortality Rate
Main general characteristics

- change of the average trend
- short-term oscillations around the average trend (risk of over-reactions)

Figure: Periodic life expectancy (left) and cohort life expectancy (right) at birth in the UK.
Life expectancy at birth in the UE.
Data and their Reliability

- Mortality analysis of a population or an insured portfolio depends on the available data and their reliability.

- These data are based on statistics coming from various national institutes (INSEE in France, Bureau of Census in the US, CMI in the UK, etc.), available through the Human Mortality Database.

- The life table is a decreasing sequence of the estimated number of people alive at date $t$ and at given age $x$ from an initial group of individuals.

- Periodic life tables, based on the mortality experience of an entire population during a relatively short period of time, usually one to three years.
• **cohort (generation) life tables** is based on mortality experience over the entire lifetime of a cohort of persons born during a relatively short period of time, usually one year.

▶ **Classical** life tables are well-suited to quantify

• **short-term mortality risk** (death insurance): 1 to 5 years if no exceptional event (ex: pandemic or heat wave).

• **BUT** these tables are not relevant for long term longevity-based contracts as mortality rates are changing over time.

• **Heterogeneity and basis risk**: the evolution of the policyholders mortality is usually different from that of the national population (selection effects).
Because of the fictitious nature of period life tables, one must be very cautious with period-based longevity indices to avoid over-reactions and as a consequence basis risk.
Heterogeneity

- Longevity patterns and longevity improvements are very different for different countries, and different geographic area.

- Factors affecting the mortality
  - socio-economic level (occupation, income, education, wealth...)
  - gender
  - marital status
  - living environment (pollution, nutritional standards, hygienic...)
Basis risk I

Difference between the national mortality data and an insured portfolio.

- Insurance companies have much more detailed information
  - They know the exact ages at death and not only the year of death (time continuous data)
  - Cause of death are specified
  - Characteristics of the policyholders: socio economic level, living conditions ...

- BUT
  - limited size of their portfolios (in comparison to national populations: 700,000 individuals from 19 different insurance companies)
Basis risk II

- small range of the observation period

Furthermore, insurers are tending to select individuals (given their health and medical history for example).

This heterogeneity is very important for longevity risk transfer based on national indices: for too important basis risk, the hedge would be too imperfect.
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   - CDB Model
   - Demography
   - Microsimulation

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Lee-Carter Model (1992)

This model describes the force of mortality, \( d_{a,t} \) at age \( a \) and time \( t \) by three series of parameters namely \( \alpha_a, \beta_a \) and \( \kappa_t \) as follows:

\[
\log d_{a,t} = \alpha_a + \beta_a \cdot \kappa_t + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim \mathcal{N}(0, \sigma),
\]

- \( \alpha_a \) gives the average level of mortality at each age \( a \) over time
- \( \kappa_t \) is the general speed of mortality improvement over time
- \( \beta_a \) is an age-specific component that characterizes the sensitivity to \( \kappa_t \) at different ages
- \( \varepsilon_{a,t} \) captures the remaining variations
- + constraints on the parameters to enforce the uniqueness: \( \sum \beta_a = 1 \) and \( \sum \kappa_t = 0. \)
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Cairns, Dowd and Blake (CDB) Model (2006)

- Different dynamics models for mortality rate by age
  - Lee-Carter Model (1992)
  - Cairns, Dowd and Blake (CDB) Model (2006). Many variants
  - For pricing purpose, models like dynamic intensity models in credit derivatives framework

- The CDB Model based on the logit function of mortality rate $q(a, t)$ ($\logit(x) = \ln \left( \frac{x}{1-x} \right)$) at age $a$ and year $t$.

  The force (intensity) of mortality is related by
  $$d(a, t) \sim \ln \left( \frac{1}{1-q(a,t)} \right)$$

  $$\logit q(a, t) = \logit(q(a, t - 1)) + \mu(a) + \sigma(a)Z, \quad Z \sim N(0, \text{Id})$$

  $$\mu(a) = \mu_1 + a \mu_2, \quad \sigma(a)Z = C^{11}Z_1 + a \ast C^{21}Z_1 + a \ast C^{22}Z_2$$

- Tests show that the model is OK for the range 30-80 years
Figure: Trend $\mu(a)$ 30-90 years (left) and Trend $\mu(a)$ 40-80 years (right)
Volatility of mortality curve

Figure: Volatility 30-90 years (left) and Volatility 40-80 years (right)
Calibration on data 1950-2006

Numerical results: age 30-90, and 40-80

\[
\begin{align*}
\mu^H &= \begin{pmatrix}
-1.491 \times 10^{-2} \\
-3.113 \times 10^{-5}
\end{pmatrix} \\
\Sigma^H &= -\begin{pmatrix}
2.488 \times 10^{-3} & -4.001 \times 10^{-5} \\
-4.001 \times 10^{-5} & 8.947 \times 10^{-7}
\end{pmatrix} \\
\mu^F &= \begin{pmatrix}
-2.208 \times 10^{-2} \\
-2.646 \times 10^{-5}
\end{pmatrix} \\
\Sigma^F &= \begin{pmatrix}
2.711 \times 10^{-3} & -4.379 \times 10^{-5} \\
-4.379 \times 10^{-5} & 9.41 \times 10^{-7}
\end{pmatrix}
\end{align*}
\]

For a 60 years women, \( \mu^F_1 \geq 10 \times (60 \mu^F_2) \): the age effect is small compared to the natural improvement.
Mortality by age and by trait

Determining factors

- Find individual characteristics (such as socio-economic level or income, educational level, postcode, marital status) that can explain mortality
- Take them into account in a stochastic mortality model

Conditional calibration

- On national mortality data and on specific data (with information on individual characteristics)
- In France, specific data=Permanent demographic sample=992711 persons, born in October only from 1866

AIM (H. Bensusan PhD Thesis): Reduce the basis risk by estimating the deviation of the ”individual mortality” from the general mean mortality given by the mortality tables.
Figure: Logit of mortality rate for French males in 2007 with different marital status
Gentlemen, get married, it’s good for you!

Figure: Males, 2007 (Top) and Males, 2017 Projection (Down)
Dynamics Microsimulation: Used by INSEE (DESTINIE) and many agencies in Australia (DYNAMOD), Canada, US, ....

- **I=The model**
  - based on a micro data base of individuals of the national data base (30,000 or + ≡ 1%)
  - input additional characteristics, as level of education, socio economic level, gender, marital status

- **II=Simulation**
  - The model takes these sample and simulates the events that occur in each individual’s life, from 1990, by stepping month-by-month through time until 2070
  - As the individuals progress through life, they experience a range of life events, in line with French data about the probabilities simulated in the model
The life events include death, fertility, divorce, emigration, level of education, labour force changes. These characteristics are often called traits.

III = Estimation
- the rate of death and birth are estimated via the CDB model.
- macroeconomic factors are also introduced and re-adjusted year by year.

IV = Dynamic Impact
- The model captures changes in demography and behaviour over time, such as an ageing population or changing fertility rates.
- Interesting for modelling the distribution of income and assets at different times, and testing the impact of public policies.
Why a new model?

- **IV = Drawbacks**
  - Simulation very too time consuming
  - leading to use very simple models

- **New Model = Same principle but**
  - No **systematic simulation** on the all sample
  - based on **random inspection** at random times
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   - Poisson Point Process
   - Individual Based Models

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Stochastic Individual-centered particle model

- **New models** in dynamics population (biology, ecology)
- **Micro-macro point of view**: micro information on individuals and large population asymptotics
- very closed to microsimulation
- **Particle model** easy to simulate
- Observations only take place at **jumps times** of Poisson process and only concern one individual each time
A Point process is a random collection of points, where each point represents the time and the characteristics of an event.

- The temporal component is enumerated by an increasing sequence $0 < T_1 < T_2 < \ldots$.
- The characteristics by a sequence $(Y_n)$ taking values in a measurable space $\mathcal{E}$.

For a Poisson point process, $(\tau_n = T_n - T_{n-1}, \ldots, Y_k)$ are mutually independent; the random variables $\tau_i \sim \mathcal{E}(\lambda)$ are iid, and $(Y_k \sim \nu)$ are also iid.

As a consequence, for any subset $A$ of $\mathcal{E}$, $N_t(A) = \text{card}\{k \leq n, Y_k \in A; T_k \leq t\}$ is a Poisson process with intensity $\lambda \nu(A)$. 
Generalized Point process

- \( N_t(A) \) is is a Poisson process with intensity \( \lambda \nu(A) \) is equivalent to the martingale property on the appropriate space of \( N_t^c(A) = N_t(A) - \lambda \nu(A) t \).

- A Generalized point process has a predictable stochastic intensity \( \mu_t(., dy) \) if for any \( A \), such that the different terms are well-defined, \( dN_t(A) - \mu_t(., A) dt \) is a local martingale.

- Links with Poisson Point Process Old result
  - For any predictable stochastic measure \( \mu_t(., dy) \), there exists some predictable process \( \psi(t, \alpha) \) defined on \( ([0, 1], d\alpha) \) such that \( \int f(y) \mu_t(.dy) = \int f(\psi(t, \alpha)) d\alpha \).
  - There exists a PPP \( N^0 \) on \( ([0, 1], d\alpha) \) such that \( \int f(t, y) N(dt, dy) = \int f(\psi(t, \alpha)))N^0(dt, d\alpha) \).
Links with Poisson Point Process More efficient simulation result

Assume $\mu_s(., dy) = k_s(y)\gamma(dy)$, and selected a iid family $(\Theta_n)$ of random numbers in $\mathbb{R}^+$

- Let $Q(dt, dy, d\theta) = \sum_{n \geq 1} \delta(T_n, Y_n, \Theta_n)(dt, dy, d\theta)$ be a Poisson process with intensity $\gamma(dy)d\theta$ on $\mathcal{E} \times \mathbb{R}^+$.

- Define a new point process $N(ds, dy)$

\[
N(ds, dy) = \int_{\mathbb{R}^+} 1_{\{\theta \leq k(s, y)\}} Q(ds, dy, d\theta) \\
= \sum_{n} \delta(T_n, Y_n)(ds, dy)1_{\{\Theta_n \leq k(T_n, Y_n)\}}
\]

- Then $\mu(s, dy)$ is the predictable intensity measure (MC Rejet method) of $N(ds, dy)$
Evolutionary Point Process

Basic Birth and Death process

- **PPProc:** \((T_n, U_n), U_n = b, e, d\) with proba \(\pi_b, \pi_e, \pi_d\),
  \[N(dt, du) = \sum_{n>0} \delta(T_n, U_n)(dt, du)\]

- **BDProc:**
  \[dZ_t = \int \phi(u) dN_t(du), \quad \phi(b) = 1, \phi(e) = 0, \phi(d) = -1\]
  is the difference of two independent Poisson processes, stopped at 0.

**EPP:** Evolutionary Point Process

- the marks \(Y_n\) are boxes (populations) with finite number of independent individuals with their own traits \(X^k\)

- additional r.v \(U_n \in \{b, e, d\}\) to express birth, nothing, death
Evolutionary Point Process

We start with a "box" with $N$ individuals with different traits.

To move from one box to an other, the algorithm is the following:

1. Pick an individual $(I_{n-1}, X_{n-1}^I)$ among the $N_{n-1}$ in the box $Y_{n-1}$.

2. Select one r.v. $U_n^I$ with a distribution depending of $(I_{n-1}, X_{n-1}^I)$ to express the evolution.

3. Select a new individual (in the universe) $(J_n, X_n^J)$ following a distribution depending of $(I_{n-1}, X_{n-1}^I)$.

4. Define the new box as the old box in which $(I_{n-1}, X_{n-1}^I)$ has been
   - replaced by $(J_n, X_n^J)$ if $U_n = e$,
   - deleted if $U_n = d$ (death)
   - retained and $(J_n, X_n^J)$ has been added if $U_n = b$ (birth)
Individual-based models of age structured population

A microscopic stochastic point model where the dynamics is specified at the level of individuals $Z_t(dx) = \sum_{j=1}^{N_t} \delta x_j(t)(dx)$

- the model takes into account trait and age of individuals,
- Add exogeneous stochastic environmental factors ($Y_t$)
- birth rate $b(x, a, Y_t^b)$, "Cairns evolution" type
- death rate $d(x, a, Y_t^d)$ "Cairns evolution" type
- $K(x', a, x) =$ distribution of trait changes of individual with age $a$ and traits $x'$

Bounded assumption on rates and $d(x, a, Y_t^d) \geq d$
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Existence and Uniqueness

Stochastic Equation driven by Poisson point process

Markovian point measures valued process

\[ Z_t(dx) = \sum_{i=0}^{N_0} \delta(X_i(Z_0), A(t,0,A_i(Z_0))) + \int_0^t \int_\epsilon Q(ds, di, d\theta, dx)1_{i \leq N_s} \]

\[ [\delta_x, A(t,s,0) 1_{0 \leq \theta \leq m_1(s,Z_{s_1},i,x)}, -\delta(X_i(Z_{s_1}), A(t,s,A_i(Z_{s_1}))) 1_{m_1(s,Z_{s_1},i,x) \leq \theta \leq m_2(s,Z_{s_1},i,x)}] \]

- \( A_i(Z_t) \) is the age of individual \( i \) at date \( t \)
- \( A(t,s,a_0) = a_0 + (t-s) \) is the age at \( t \) of individual with age \( a_0 \) at \( s \)

\[ m_1(s,Z_{s_1},i,x) = b \left( X_i(Z_{s_1}), A_i(Z_{s_1}), Y_s^b \right) k \left( X_i(Z_{s_1}), A_i(Z_{s_1}), x \right) \]

\[ m_2(s,Z_{s_1},i,x) = m_1(s,Z_{s_1},i,x) + d \left( X_i(Z_{s_1}), A_i(Z_{s_1}), Y_s^d \right) \times k \left( X_i(Z_{s_1}), A_i(Z_{s_1}), x \right) \]
Macroscopic approximation for large population

Large numbers asymptotic

- Large numbers renormalization + weakly convergence of initial population $\frac{1}{n} \sum_{i=0}^{N_n(0)} \delta(X_i^n(0), A_i^n(0))$

- Uniform Moments assumption

Limit process:

- Conditional (w.r.t.Y) deterministic non-linear PDE
- Give Information on long term behavior
- Help to understand the microscopic point of view
McKendrick (1926) and VonFoerster (1959) : Density PDE in demography

\[
\left( \frac{\partial g}{\partial t} + \frac{\partial g}{\partial a} \right)(a, t) = -d(a)g(a, t), \quad g(0, t) = \int_0^{\infty} b(a)g(a, t)da
\]

Approximation (a.s) for large populations with stochastic factors \((Y^b, Y^e, Y^e)\)

\[
\left( \frac{\partial g}{\partial t} + \frac{\partial g}{\partial a} \right)(\omega, x, a, t) = - \left[ d(x, a, Y^d_t) + e(x, a, Y^e_t) \right] g(x, a, t)
\]

\[
+ \int_{\chi} e(x', a, Y^e_t)k^e(x', a, x)g(., x', a, t)\gamma(dx')
\]

\[
g(\omega, x, 0, t) = \int_{\tilde{\chi}} b(x', a, Y^b_t)k^b(x', a, x)g(\omega, x', a, t)\gamma(dx')da
\]

\[
g(\omega, x, a, 0) = g_0(\omega, x, a),
\]
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Framework

- whole life annuity-immediate, of 1 unit payable at the end of each year while the life aged \( a \) survives
- Sample of \( c = 10,000 \) policies
- Run micro-macro model and estimate the future cash flows (estimated reserve)
- Make future projection

Strategic for risk management and regulation

- integrated analysis on demographic and financial risk
- a line to find operative links between the risk parameters in the model and minimal solvency margin required by Solvency III
Statistics for the evolution: female

Figure: Cash flows of a life annuities portfolio for 60 years old French female considered by central scenario and confidence interval.
Figure: Cash flows of a life annuities portfolio for 60 years old French male considered by central scenario and confidence interval
Figure: Cash flows of a life annuities portfolio for 60 years old French male considered by central scenario for different marital status
Figure: Cash flows of a life annuities portfolio for 60 years old French male considered by central scenario for different education levels.
Basis Risk on real life annuity portfolio

Real Portfolio of Life insurer

Figure: Age Distribution of insured
Projection in Ten years of real life annuities portfolio

Figure: Cash flows of a life annuities in 2009 on real portfolio
Some evolution scenarios for real portfolio

Figure: Scenarios of evolution real portfolio in 10Y and 20Y
Nominal Chooser Swaption

Structured Swap to transfer only inflation and interest rate risk (Harry’s Idea)

▶ Give to the insurer the opportunity to fix the series of nominaux of the future swap

▶ To be able to face on uncertainty on the estimation, she can propose 2 curves: one more conservative than the other, (using micro-macro modeling for instance).

▶ At the starting date of the swap, she has the opportunity to adjust her anticipations by entry in a convex combination of the 2 given curves.

▶ That is the optional part of the contract

For the bank, no real longevity risk, but a strong counter part risk
This conference aims at confronting different viewpoints on stochastic models of future human longevity: demographers, geneticians, epidemiologists of ageing, actuaries, statisticians,

This 2-day event is part of the special research semester on longevity organized by the research chairs of the Institut Louis Bachelier.

Confirmed speakers include S.Austad (Purdue), H.Bensusan (Soc. Gen), V.Canudas-Romo (Tom Hopkins U.), S. Gaille (Lausanne), S.Haberman (Cass Business School), R.Norberg, N.Ouellette (Berkeley), E. Pitacco (Trieste), F.Planchet (ISFA), R.Rau (Max Planck Institute), Jean-Marie Robine (INSERM), Yahia Salhi (SCOR & ISFA), M. Sherris (Sydney).